WAVE OPTICS

Syllabus

Unit	Learning Units	Lecture Hours
I	**Interference of light: (Problem)** **A) Division of Wavefront:** Introduction, Conditions for the interference of light, Interference of light by division of wavefront and amplitude, Phase change on reflection- Stokes' treatment, Fresnel's Bi-Prism-Determination of Wavelength of Light. **B) Division of Amplitude**: Cosine law - colours in thin films, Newton's rings in reflected light -Theory and experiment - Determination of wavelength of monochromatic light, Michelson interferometer and determination of wavelength.	12
II	**Diffraction of light(Problem)** **A) Fraunhofer Class**: Distinction between Fresnel and Fraunhofer diffraction, Fraunhofer diffraction at a single slit, double slit and N-slits (No Derivation for N-Slits), Determination of wavelength of light using a diffraction grating, Resolving power of grating, **B) Fresnel's Class**: Fresnel's half-period zones, Zone plate, comparison of zone plate with a convex lens.	12
III	**Polarisation of light(Problem)** **A) Polarized light**: Methods of production of plane-polarized light - Polarisation by reflection (Brewster's law), Malus law, Double refraction, Nicol prism, Nicol prism as polarizer and analyzer, Quarter wave plate, Half wave plate **B) Types and production of polarized Light**: Plane, Circularly and Elliptically polarized light-Production and detection, Optical activity, Laurent's half shade polarimeter: determination of the specific rotation	12

IV	**A) Aberrations**: **(Problem)** Monochromatic aberrations - Spherical aberration, Methods of minimizing spherical aberration, Coma& Astigmatism - minimization methods, Chromatic aberration-the achromatic doublet; Achromatism for two lenses (i) in contact and (ii) separated by a distance. **B) Fibre Optics:(No Problem)** Fibre optics: Introduction to Fibers, different types of fibers, rays and modes in an optical fiber, Principles of fiber communication (qualitative treatment only), Advantages of fiber optic communication.	12
V	**Lasers and Holography (No Problem)** **A) Lasers**: Introduction, Spontaneous emission, stimulated emission, Population Inversion, Laser principle, Einstein coefficients, Types of lasers-He-Ne laser, Ruby laser, Applications of lasers; **B) Holography**: Basic principle of holography, Applications of holography	12

Text BOOKS:

- BSc Physics, Vol.2, Telugu Akademy, Hyderabad
- Unified Physics Vol.II Optics, Jai PrakashNath & Co.Ltd., Meerut., Meerut

REFERENCE BOOKS:

1. A Text Book of Optics-N Subramanyam, L Brijlal, S.Chand &Co.
2. Optics-Murugesan, S. Chand & Co.
3. Optics, F.A. Jenkins and H.G. White, McGraw-Hill
4. Optics, Ajoy Ghatak, Tata McGraw-Hill.
5. Introduction of Lasers – Avadhanulu, S. Chand &Co.
6. Principles of Optics- BK Mathur, Gopala Printing Press,1995

UNIT I INTERFERENCE

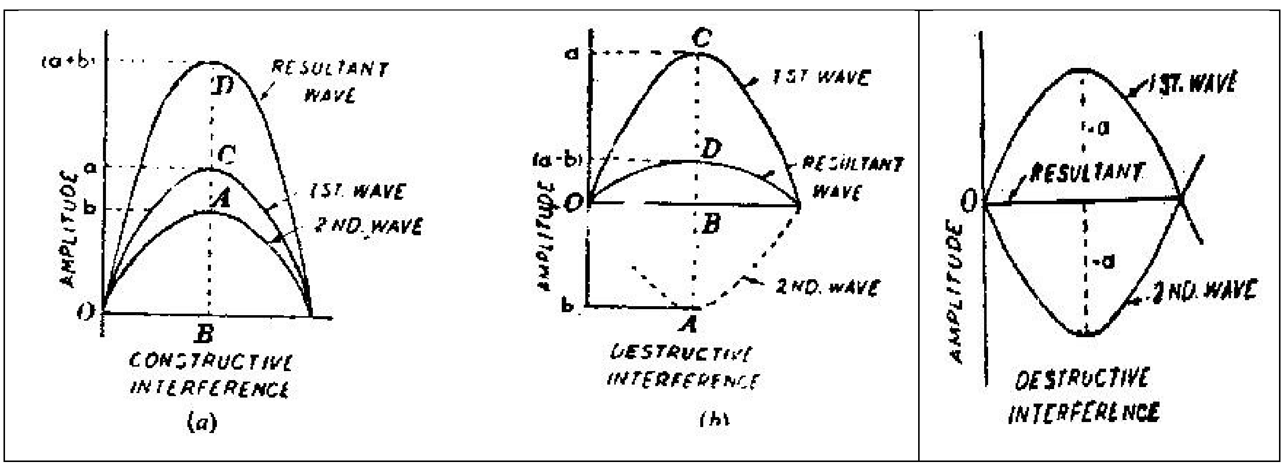
Interference: when two light waves of same frequency and having constant phase difference, coincide in space and time, there is a modification in the intensity of light. This *modification in the intensity of light due to superposition of two waves is called interference.*

The interference pattern in which the position of maxima and minima of intensity of light remains constant is called sustained interference.

Principle of superposition: when two light waves arrive at a point in space simultaneously, the net wave disturbance at that point and at any given time is the vector sum of all the wave disturbances at that point at that particular time. This is called the principle of superposition.

Suppose the two waves are of the same frequency but are of different amplitudes, say a and b where a>b. if they reach a certain point in phase with each other as in fig(a), then the resultant displacement or amplitude is equal to the sum of the two amplitudes i.e., (a+b). The two waves reinforce each other and are said to produce constructive interference.

However, as shown in fig (b) if the two waves reach π radians or 180^0 out of phase with each other, then the resultant amplitude is equal to the difference of the two amplitudes .i.e.,(a-b). If a=b then the resultant amplitude is zero as shown in fig (c). The two waves neutralize each other and are said to produce destructive interference.



Conditions necessary for permanent interference:

1) The two sources must emit continuous waves of the same wavelength and time-period.
2) The waves emitted by the two sources must start either exactly in phase or with a constant phase difference. This can be possible only if the two sources are derived from a single parent sources.

3) The two coherent sources must be close to each other.

4) The two coherent sources must be extremely small.

5) The amplitudes of the two waves should preferably be equal.

6) The two coherent sources should emit monochromatic light.

7) The two waves must be propagated along the same direction to get coincidence.

Coherent sources: Two sources are said to be coherent if they emit light waves of same frequency, nearly same amplitude and are always in phase with each other.

We can produce coherent waves in two ways:

1. Division of wave front
2. Division of amplitude

1) Division of wave front: when light from a source is allowed to pass through two slits, the original wave front gets divided into two. These two slits will act as coherent sources and emit coherent waves.

2) Division of amplitude: The original light beam is divided by partial reflection from a glass plate or at the surface of a thin film. The refracted part and the reflected part are coherent.

Analytical treatment of Interference:

Consider two waves of same frequency and amplitudes a_1 and a_2 respectively. Let y_1 and y_2 be the displacements of two waves

Suppose $y_1 = a_1 \text{Sin}\,\omega t$

and $y_2 = a_2 \text{Sin}\,(\omega t + \Phi)$ where Φ is the phase difference between the two waves at the point under consideration.

According to principle of superposition, the resultant displacement of the particle is given by,

$$y = y_1 + y_2 = a_1 \text{Sin}\,\omega t + a_2 \text{Sin}\,(\omega t + \Phi)$$

$$= (a_1 + a_2 \text{Cos}\Phi)\, \text{Sin}\omega t + a_2 \text{Sin}\Phi\, \text{Cos}\,\omega t \qquad \text{----} \quad (1)$$

let $a_1 + a_2 \text{Cos}\Phi = A\, \text{Cos}\delta$ --- (2) and

$a_2 \text{Sin}\Phi = A\, \text{Sin}\delta$ --- (3)

Substitute eq(2) and eq(3) in eq(1) then,

$y = A\, \text{Cos}\delta \text{Sin}\omega t + A\, \text{Sin}\delta\, \text{Cos}\,\omega t = A\, \text{Sin}\,(\omega t + \delta)$

Where, A is the amplitude of the resultant disturbance at that point.

Intensity of the resultant disturbance, $I = A^2 = (a_1 + a_2 \text{Cos}\Phi)^2 + a_2^2\, \text{Sin}^2\Phi$

$$= a_1^2 + a_2^2 + 2\, a_1 a_2 \text{Cos}\Phi$$

$$= a^2 + a^2 + 2a^2 \text{Cos}\Phi \qquad \text{if } a_1 = a_2 = a$$

$$= 2a^2\,[1 + \cos\Phi] = 2a^2 \,.\, 2\cos^2(\Phi/2)$$

$$= 4a^2 \cos^2(\Phi/2) \qquad --- \qquad (4)$$

Case (1): - Constructive Interference

When the phase difference $\Phi = 0, 2\pi, 4\pi, ------, 2\pi n$ then,

$$I = A^2 = a_1^2 + a_2^2 + 2\,a_1a_2 = (a_1 + a_2)^2$$

This is called the constructive interference in which the resultant intensity is maximum.

If $a_1 = a_2$, then $I = A^2 = 4a_1^2$

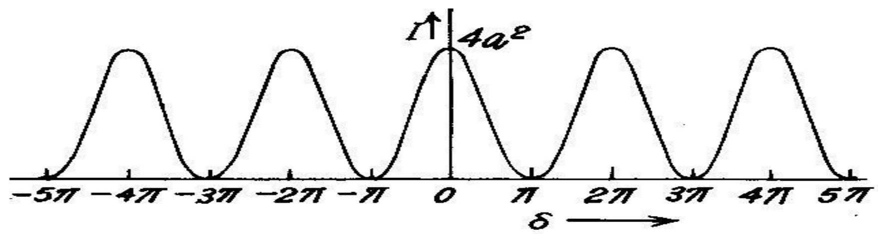
Case (2): - Destructive interference

When the phase difference $\Phi = \pi, 3\pi, ------, (2n + 1)\,\pi$ then,

$$I = A^2 = a_1^2 + a_2^2 - 2\,a_1a_2 = (a_1 - a_2)^2$$

This is called the destructive interference in which the resultant intensity is minimum.

If $a_1 = a_2$, then $I = 0$



Phase change on reflection:

According to Stoke's, when a light wave is reflected at the surface of an optically denser medium it suffers a phase change of 'π' i.e., a path difference of $\lambda/2$, No phase change is introduced if the reflection takes place from the surface of rarer medium.

Let us consider a wave AO of light of amplitude '*a*' be incident at point O on the boundary M_1M_2 of the media I and II. Medium - II is optically denser than medium – I. The wave is partly reflected along OB and partly transmitted. Let '*r*' be the reflection co-efficient and '*t*' be transmission co-efficient. Now the amplitude of reflected wave will be '*ar*' while that of transmitted wave is '*at*'

Now suppose that the directions of reflected and transmitted rays are reversed at shown in Fig (B). By doing so we should have a wave 'OA' of original amplitude '*a*'

On reversing OB we get a reflected wave OA of amplitude ($ar \times r$) 'ar^2' and transmitted wave OD of amplitude art. Let r' and t' be the r' and t' be the fractions of amplitude reflected and transmitted when the ray is travelling from denser to rarer medium. Now on reversing CO we get a reflected wave OD of amplitude atr' and transmitted wave of amplitude ($at \times t'$) att'. As there was no wave originally along OD hence OD should be zero.

$\therefore art + atr' = 0 \Longrightarrow r + r' = 0 \Longrightarrow r = -r'$ ------------(1)

Along OA there should be a wave of amplitude 'a'

$$ar^2 + att' = a$$

$$\Rightarrow r^2 + tt' = 1$$

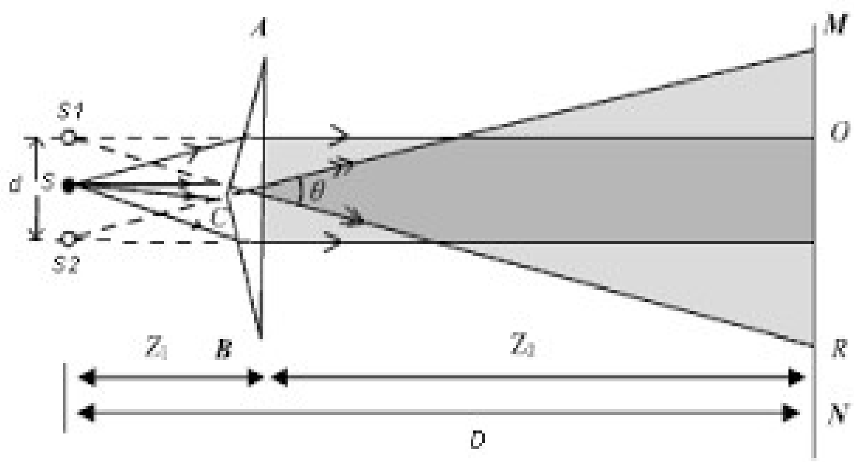
$$\Rightarrow tt' = 1 - r^2$$

The -ve sign in Eq (1) indicates a phase change 'π' either at reflection from rarer to denser medium or at reflection from denser medium to rarer medium.

Fresnel's biprism:

A Fresnel Biprism is a thin double prism placed base to base and have very small refracting angle (0.5°). This is equivalent to a single prism with one of its angles nearly 179° and other two of 0.5° each.

The interference is observed by the division of wave front. Monochromatic light through a narrow-slit S falls on biprism, which divides it into two components. One of these components is refracted from upper portion of biprism and appears to come from S_1 where the other one refracted through lower portion and appears to come from S_2. Thus, S_1 and S_2 act as two virtual coherent sources formed from the original source. Light waves arising from $S_1$and S_2 interfere in the shaded region and interference fringes are formed which can be observed on the screen.



Application of Fresnel's Biprism

Fresnel biprism can be used to determine the wavelength of a light source (monochromatic), thickness of a thin transparent sheet/ thin film, refractive index of medium etc.

Determination of wave length of light

As expression for fringe width is $\beta = \dfrac{\lambda D}{d}$

Biprism can be used to determine the wavelength of given monochromatic light using the expression. $\lambda = \frac{\beta d}{D}$

Experimental Arrangement:

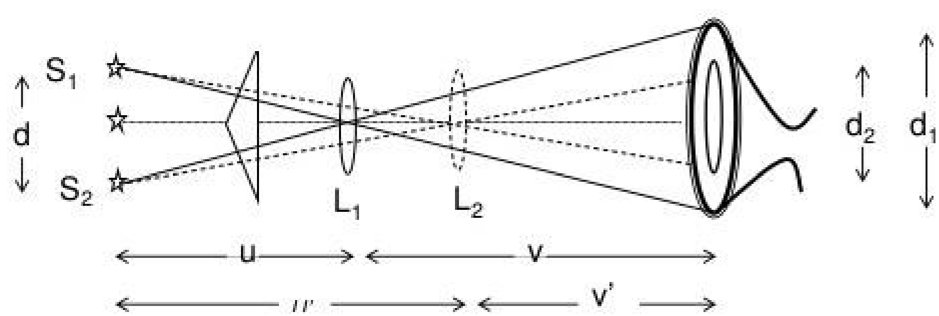
Light from the monochromatic source is made to fall on a thin slit mounted vertically on a rigid optical bench fitted with a scale. The biprism and the screen (in this case an eye piece) are also mounted vertically. The eye piece can be moved in the plane perpendicular to the axis of bench using a micrometre-based translation stage.

(i) Measurement of fringe width: To get β, fringes are first observed in the field of view of the microscope. The vertical wire of the eyepiece is made to coincide with one of the fringes and screw of micrometer is moved sideways and the number of fringes is counted.

β =Distance moved / number of fringes passed

(ii) Measurement of D: This distance between source and eyepiece is directly measured on the optical bench scale.

(iii) Determination of d: To determine the separation between the two virtual sources (d), a convex lens of short focal length is introduced between the biprism and the eyepiece, keeping the distance between the slit and eyepiece to be more than four times the focal length of lens. The lens is moved along the length of the bench to a position where two images of slits are seen in the plane of cross wires of eye piece. The distance between these two images of the slit is measured by setting the vertical cross wire successively on each of images and recording the two positions of the cross wire using micrometre. Let this separation be d_1. Now the lens is moved such that for another position of the lens, again two images of the slit are seen on the eyepiece. Let d_2 be the separation between these two images.



$$\frac{d_1}{d} = \frac{u}{v} \qquad (1)$$

$$\frac{d2}{d} = \frac{v'}{u'} \qquad (2)$$

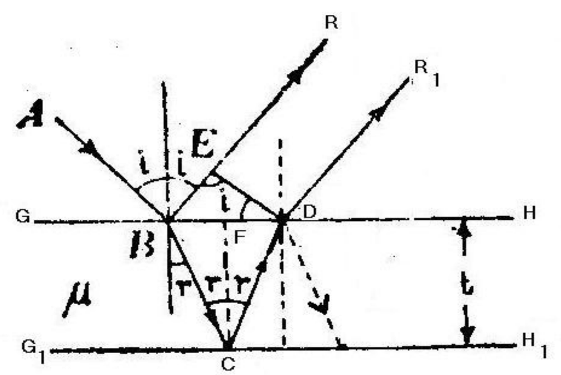
Since these two positions of lens are conjugate, the separation between the virtual source 'd' is given by using equations 1 and 2 as

$d = \sqrt{d_1 d_2}$ where d_1 and d_2 are the distance between S_1 and S_2 for two positions of the lenses.

Colors of Thin films (Reflected light): (Cosine Law)

Colors of thin films like soap bubbles, oil layers on water etc., are a consequence of interference of light beams reflected from the films. The two coherent beams are produced by the division of the amplitude of the incident light beam. A film is said to be thin when its thickness is about 5000 A^0.

Let GH and G_1H_1 be the two surfaces of a transparent film of uniform thickness t and refractive index μ as shown in fig. Suppose a ray AB of monochromatic light be incident on its upper surface. This ray is partially reflected along BR and refracted along BC. After one reflection at C, we obtain the ray CD. After refraction at D,



the ray finally emerges out along DR_1 in air. Obviously, DR_1 is parallel to BR. Our aim is to find out the effective path difference between the rays BR and DR_1. For this purpose, we draw a normal DE on BR.

The path difference Δ is given by, $\Delta = \mu\,(BC + CD) - BE$ --- (1)

From fig, CF / BC = Cos r or t / BC = Cos r

∴ BC = CD = t / Cos r --- (2)

To calculate BE, we first find BD which is equal to (BF + FD). We consider triangle BFC

BF / FC = tan r or BF / t = tan r

∴ BF = t tan r

Now BD = BF + FD = 2BF = 2t tan r (∵ BF = FD)

From triangle BED, BE / BD = Sin i

or BE = BD Sin i = 2t tan r Sin i

we know that

$$\sin i / \sin r = \mu \quad \text{or} \quad \sin i = \mu \sin r$$

$$BE = 2t \tan r \, (\mu \sin r)$$

or $$BE = 2\mu t \tan r \sin r \quad \text{---} \quad (3)$$

from eqs.(2) and eqs.(3), substituting the values of AB and BE in eq(1). We get

$$\Delta = \mu\,(2t / \cos r) - 2\mu t \tan r \sin r$$

$$= \frac{2\mu t}{\cos r} - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}\,[1 - \sin^2 r] = \frac{2\mu t}{\cos r} \cos^2 r$$

$$= 2\mu t \cos r \quad \text{---} \quad (4)$$

For the net optical path difference, we have to consider the phase change of π occurred while the first reflected ray BR reflects from denser to rarer medium. i.e., the first reflected ray experiences an extra path difference of $\lambda/2$.

$\therefore$ Net optical path difference = $2\mu t \cos r \pm \lambda/2$ --- (5)

Condition for bright fringe: $2\mu t \cos r \pm \lambda/2 = n\lambda$

Condition for dark fringe: $2\mu t \cos r \pm \lambda/2 = (2n + 1)\lambda/2$

Colors in thin films

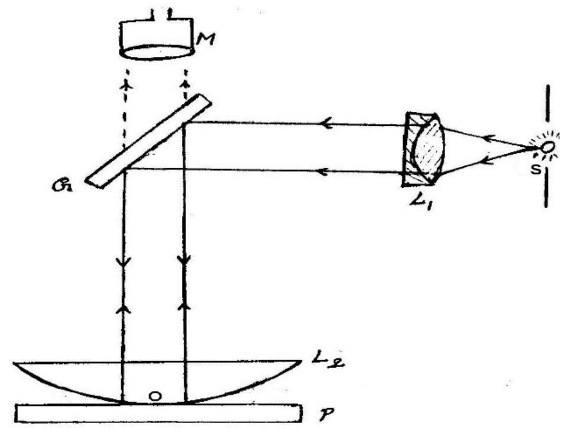
These colors are due to interference between light waves reflected from the top and the bottom surfaces of thin films. When white light is incident on a thin film, the film appears coloured and the color depends upon the thickness of the film and also the angle of incidence of the light.As a result, we observe colors in the order of violet, blue....... etc., If the angle of refraction is changed, we will get different colors for different angles.

Newton's Rings:

A Plano-convex lens with its convex surface is placed on a plane glass plate; an air film of gradually increasing thickness is formed between the two. The thickness of the film at the point of contact is zero. If monochromatic light is used to fall normally and the film is viewed in reflected light, alternate dark and bright rings concentrated around the point of contact between the lens and glass plate are seen. This phenomenon was first described by Newton hence the rings are known as Newton's rings.

Experimental arrangement:

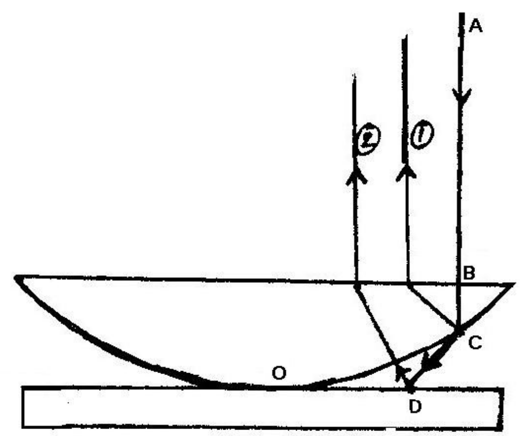
S is an extended monochromatic source whose light is made parallel by lens L_1. This light falls on a glass plate G at 45^0 and is therefore reflected normally on a Plano- convex lens L_2 placed on a flat glass plate P. Light is reflected upwards from the upper and lower faces of the wedge-shaped air film enclosed between the curved surface of L_2 and P. These two reflected rays interfere and give rise



to an interference pattern in the form of circular rings.The circularity of fringes is due to the fact that air film is symmetrical about the point of contact O. The diameter of these rings can be measured with the help of a traveling microscope M placed vertically above L_2.

Explanation of formation of Newton's rings:

The formation of Newton's rings can be explained with the help of fig. AB is a monochromatic ray of light, which falls on the system. A part is reflected at C, which goes out in the form of ray 1 without any phase reversal. The other ray is refracted along CD. At point D it is again reflected and goes out in the form of ray 2 with a phase reversal of π. The reflected rays 1 and 2 are in a position to produce



interference. As the rings are observed in reflected light, the path difference between them is ($2\mu t \cos r + \lambda/2$).

For air film $\mu = 1$ and for normal incident $r = 0$.

Path difference = $2t + \lambda/2$

At the point of contact $t = 0$

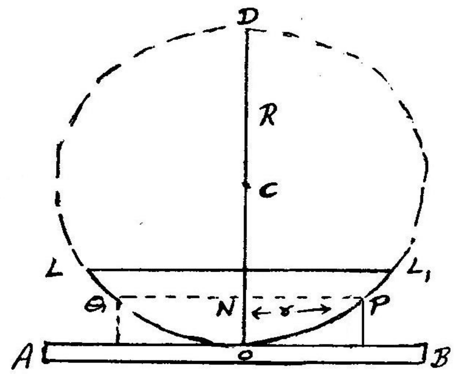
Path difference = $\lambda/2$

Which is the condition of minimum intensity. Thus the central spot is dark.

For n^{th} maximum we have, $2t + \lambda/2 = n\lambda$ --- (1)

Calculation of diameter of bright and dark rings:-

Let LOL_1 be the lens placed on a glass plate AB. The curved surface LOL_1 is the part of spherical surface with center at C. Let R be the radius of curvature and r be the radius of Newton's rings corresponding to the constant film thickness t.



From eq(1) $2t = (2n - 1)\lambda/2$ --- (2) for the bright ring where n =1,2,3.......

and $2t = n\lambda$ --- (3) for dark ring where n = 0,1,2,3.....

From the property of the circle, $NP \times NQ = NO \times ND$

Substitute the values, $r \times r = t(2R - t)$

$= 2Rt - t^2$

$\approx 2Rt$ [$\because$ t^2 is negligible]

$\therefore$ $r^2 = 2Rt \implies t = r^2 / 2R$ --- (4)

For a bright ring,

Substitute (4) in (2) $\implies$ $2r^2 / 2R = (2n - 1)\lambda/2$

or $r^2 = R(2n - 1)\lambda/2$

$r = \sqrt{R(2n - 1)\lambda/2}$

or $d = \sqrt{2\lambda R(2n - 1)}$ --- (5)

or $d \propto \sqrt{(2n - 1)}$

The diameter (or radius) of the bright rings is proportional to the square root of odd natural numbers.

Similarly for a *dark ring*,

Substitute (4) in (3) $\implies$ $2r^2 / 2R = n\lambda$

or $r^2 = n\lambda R$

$r = \sqrt{n\lambda R}$

or $d = 2\sqrt{n\lambda R}$

or $d \propto \sqrt{n}$

The diameter (or radius) of the dark rings is proportional to the square root of natural numbers.

Note : The fringe width decreases with the order of the fringe and fringes get closer with increase in their order.

Determination of wavelength (λ) of sodium light using Newton's rings:-

The diameter of n^{th} dark ring is, $d_n^2 = 4n\lambda R$ --- (i)

The diameter of $(n + m)^{th}$ dark ring is, $d_{n+m}^2 = 4(n + m)\lambda R$ --- (ii)

Equation (ii) – equation (i) ➔ $d_{n+m}^2 - d_n^2 = 4m\lambda R$

$$\therefore \quad \lambda = \frac{d_{n+m}^2 - d_n^2}{4mR}$$

Similar results can be obtained by taking bright rings.

The diameters d_n and d_{n+m} corresponding to n^{th} and $(n+m)^{th}$ dark rings are noted with traveling microscope and radius of curvature of the lens is measured with a spherometer (by using the formula, $R = \frac{l^2}{6h} + \frac{h}{2}$ where, l – distance between two legs of the spherometer and h – difference of the readings of the spherometer when it is placed on the lens as well as when placed on the plane surface). So the wavelength of the light can be found out.

Determination of Refractive index (μ) of a liquid:- If there is a air film between the plano – convex lens and the plane glass plate. The diameter of n^{th} and $(n + m)^{th}$ dark ring are measured with traveling microscope. Then $d_{n+m}^2 - d_n^2 = 4m\lambda R$ --- (iii)

If the air film is replaced with liquid, the diameter of n^{th} and $(n + m)^{th}$ dark rings are

$$(d^I_{n+m})^2 - (d^I_n)^2 = 4m\lambda R / \mu \quad \text{---} \quad (iv)$$

Substitute eq(iii) in eq(iv) we get,

$$(d^I_{n+m})^2 - (d^I_n)^2 = (d_{n+m}^2 - d_n^2) / \mu$$

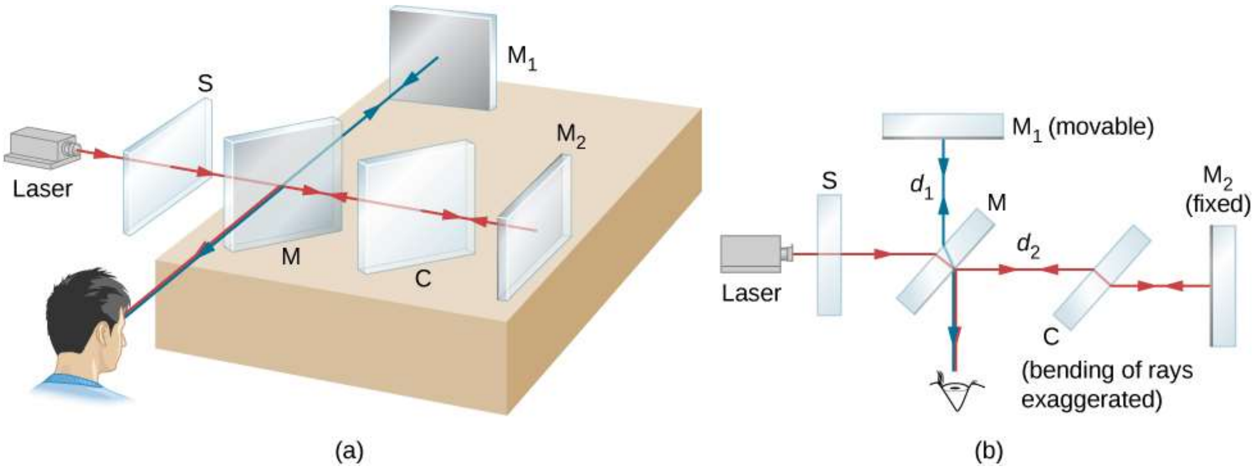
$$\Longrightarrow \quad \mu = \frac{d_{n+m}^2 - d_n^2}{d'^2_{n+m} - d'^2_n} \quad \text{---} \quad (v)$$

Therefore, from the above equation we can calculate the refractive index of a liquid.

Uses of Newton's Rings – we can calculate the refractive index (μ) of liquid, Radius of curvature (R) of given lens and wavelength (λ) of light.

Describe the construction of Michelson interferometer and explain its working. How is wavelength of monochromatic light determined with it?

Michelson's interferometer consists of highly polished mirrors M_1, and M_2 which are at right angles to each other. G_1 and G_2 are two optically fiat glass plates of same thickness and of same material, placed parallel to each other. These plates are inclined at an angle of 45° with the mirrors M_1 and M_2. The face of G, towards G_2 is semi silvered. The mirror-M_1 is mounted on a carriage which can be moved forward or backward. The mirrors M_1 and M_2 are provided with three levelling screws with the help which they can be tilted about horizontal and vertical axes. A telescope 'T' receives the reflected light from the mirrors M_1 and M_2 Working Light from a monochromatic source 'St after being rendered parallel by a collimating lens L falls on the semi silvered glass plate G_1. It is divided into two parts, one being reflected from the semi-silvered surface of G_1 [ray (1)] which travels towards M_1, and the other being transmitted giving rise to the ray (2), which travels towards M_2. The two rays fall normally on M_1 and M_2 and are reflected along their original paths. The reflected rays again meet at G_1 and enter the telescope T. The two rays are originally derived from the same single beam hence they can produce interference fringes in the field of view of the telescope. Ray No. 1 passes through G_1 twice whereas ray No. 2 does not do so even once. Hence the two paths OM_1 and OM_2 are not equal in the absence of G_2. To equalise the paths, a glass plate G_2 of same thickness and of same material as that of G_1, is introduced in the path of ray No.2. Hence this is called as compensating plate. Looking in the direction M_1 from E one observes M_1, and a virtual image M_2' of M_2. Thus, the interference fringes may be conceded as to be formed by light reflected from the surface of M_1 and M_2' respectively. Thus, the arrangement is equivalent to air film enclosed between the reflecting surfaces M_1 and M_2'. The interference fringes may be straight, circular, parabolic, etc. depending upon path difference and the angle between mirrors M_1^I and M_2^I.



Determination of wavelength of mono-chromatic light: First of all, the Michelson's interferometer is set for circular fringes with central bright spot. If t be the thickness of the air film enclosed between the two mirrors and n the order of the spot obtained, the as for normal incidence cos r = 1, we have

$$2t + \frac{\lambda}{2} = n\lambda$$

If now M_1 is moved λ/2 away from M_2', then an additional path difference of λ will be introduced any hence (n+1)th bright spot appears at the centre of the field. Thus, each time when M_1 moves through a distance λ/2, next bright spot appears at the centre of the field. Let N be the number of fringes that cross the centre of field when the mirror M_1 is oved from initial position x_1 to a final position x_2, then,

$$N\frac{\lambda}{2} = x_2 - x_1 \Rightarrow \lambda = \frac{2(x_2 - x_1)}{N}$$

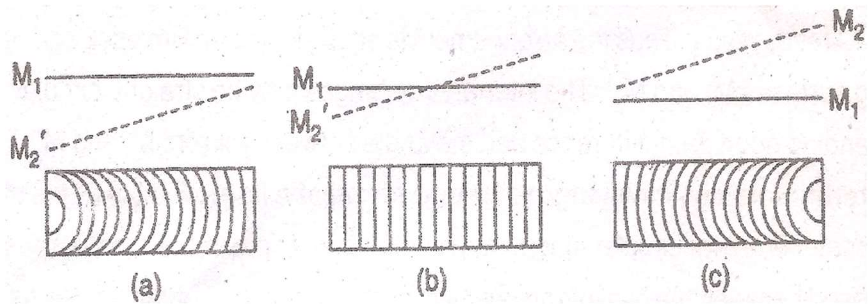
The difference ($x_2 - x_1$) is measured with micrometer screw and N is counted. The experiment is repeated at several times and the mean values of λ is obtained.

Explain the types of fringes formed in Michelson's interferometer.

1) when mirror M_1 and the virtual mirror M_2^I (image of M_2) are parallel, circular fringes are formed.
2) When M_1 and M_2^I coincide, the path difference is zero & the field of view is perfectly dark.
3) when M_1 and M_2^I are inclined, the air film enclosed is wedge-shaped & straight-line fringes are observed.

   If M_1 intersect M_2^I in the middle, the fringes are perfectly straight.

In other positions, the shape of the fringes is as shown in fig. They are curved & are always concave towards the thin edge of the wedge.



Explain the uses of Michelson's interferometer.

Michelson's interferometer has been used for,

1. In the determination of wavelength of monochromatic source of light,
2. To determine the difference between the two neighborhood wavelengths or resolution of the spectral lines.
3. In the determination of refractive index and thickness of various thin transparent materials.
4. For the measurement of the standard meter in terms of the wavelength of light etc.

1. Determination wavelength of mono-chromatic light: First, the Michelson's interferometer is set for circular fringes with central bright spot. Let *t* be the thickness of the air film enclosed between the two mirrorsand n he the order of the spot obtained, for normal incidence cos r = 1, we have

$$2t+\frac{\lambda}{2}=n\lambda$$

If now M_1 is moved λ/2 away from M_2', then an additional path difference of λ will be introduced and hence (n+1)th bright spot appears at the centre of the field, Thus, each timewhen M_1 moves through a distance λ/2, next bright spot appears at the centre of the field. Let N be the number of fringes that cross the centre of field when the mirror M_1 is moved from initial position x_1 to a final position x_2, then,

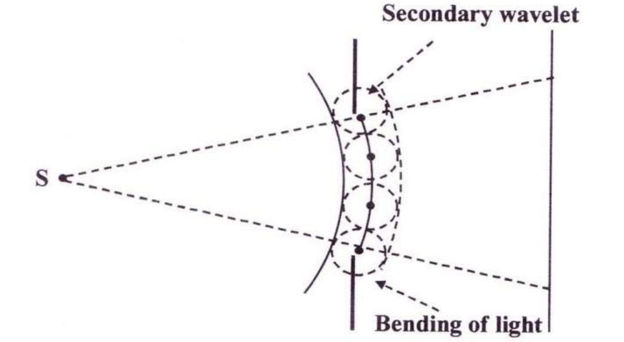
$$\lambda=\frac{2(x_2-x_1)}{N}$$

The difference (x_2-x_1) is measured with micrometer screw and N is counted. The experiment is repeated at several times and the mean value of λis obtained.

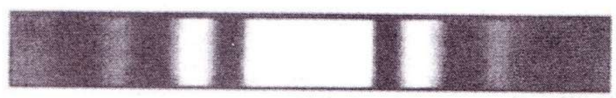
UNIT-II
Diffraction of light

The bending of light around the edges of an obstacle or the encroachment of light within the geometrical shadow area is called diffraction.

For diffraction to occur the size of the object must be of the order of the wavelength of the incident waves; when the wavelength (of light is ~ 5×10^{-5} cm) is much smaller than the size of the object, diffraction is ordinarily not observed, and the object casts a sharp shadow. Diffraction pattern consists of bright and dark bands like the interference pattern and the bands thus formed are known as diffraction bands or fringes.



Bending of light round the edges and Diffraction pattern



The correct interpretation of diffraction of light was given by Fresnel based on the wave theory of light.

Diffraction phenomena are a part of our common experience. The luminous border that surrounds the profile of a mountain just before the sun rises behind it, the light streaks that one sees while looking at a strong source of light with half shut eyes and the colour spectra that observes while viewing a distant source of light through a fine piece of cloth are all examples of diffraction effects.

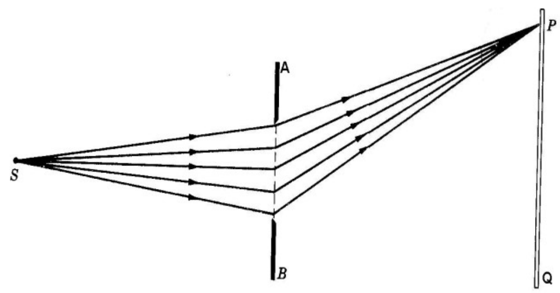
<u>Differences between Interference and diffraction: -</u>

1) In the phenomenon of interference, the interaction takes place between two separate wave fronts originating from the two coherent sources while in the phenomenon of diffraction the interaction takes place between the secondary wavelets originating from different points of the exposed parts of the same wave front.
2) In the interference pattern the region of minimum intensity are usually almost perfectly dark while it is not so in diffraction pattern.
3) The width of the fringes in interference may or may not be equal or uniform. While in diffraction pattern fringe width of various fringes are never equal.
4) In an interference pattern all the maxima are of same intensity but in a diffraction pattern they are of varying intensity.

There are ***two types of diffraction.***

(**i**) Fresnel diffraction and (**ii**) Fraunhofer diffraction.

Fresnel diffraction:- In this type, the source, or the screen or both are at finite distances from the aperture or obstacle causing diffraction. In this type of no lenses are used.

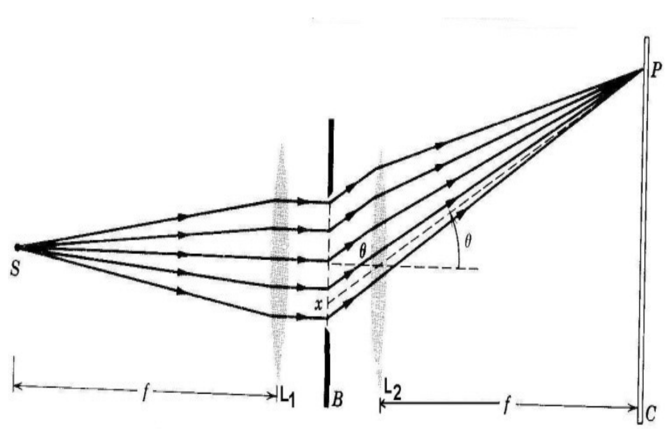


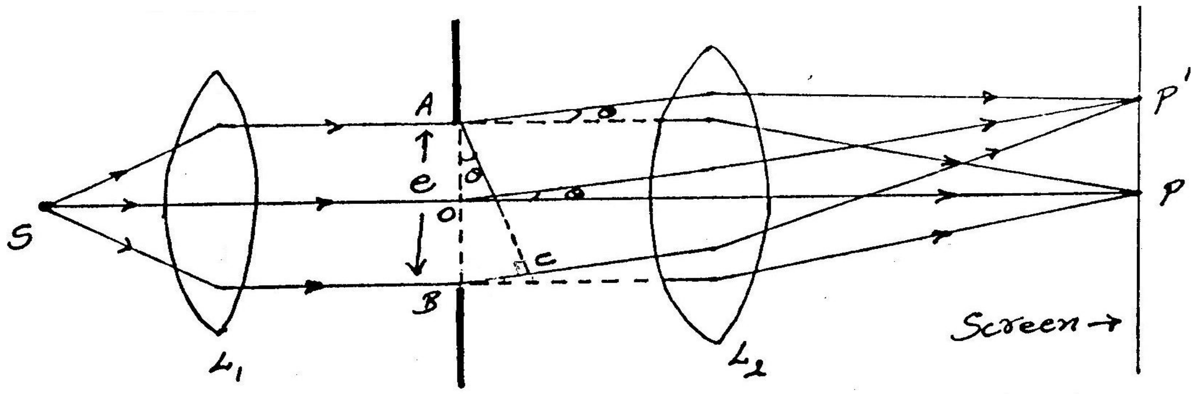
Fraunhofer diffraction: - In this type, the source, and the screen on which the pattern is observed are at infinite distance from the aperture or obstacle causing diffraction. In this type lenses are used. The incoming light is rendered parallel with a lens and the diffracted beam is focused on the screen with another lens.

Out of these two types, Fraunhofer diffraction is simple to understand. We can easily produce Fraunhofer diffraction in the laboratory using a spectrometer.

<u>Diffraction at a single slit :-</u>

A slit is a rectangular aperture whose length is large compared to its breadth. Let 'e' be the width of slit AB. S is a source of light. Lenses L_1 and L_2 are used for collimation and convergence purposes. P is the point image of S surrounded by alternate maxima and minima. The un-diffracted and direct light is converged at P with maximum intensity.





Let us consider secondary wavelets from AB move in a direction OP^1 inclined at an angle θ with respect to un-deviated light. The point P^1 is of the minimum intensity depending upon the path difference between the secondary waves originating from the corresponding points of the wavefront. To find out the intensity at P^1, draw a normal AC on BP. The path difference between secondary wavelets from A and B in direction 'θ'

$$= BC = AB \sin\theta = e \sin\theta$$

and corresponding phase difference = $2\pi e \sin\theta / \lambda$

Let us consider that the width of the slit is divided into N equal parts and the amplitude of the wave from each part is a. The phase difference between any two consecutive waves from these points would be,

$$\frac{1}{n}(\textit{total}\ \text{phase}) = \frac{1}{n}\left(\frac{2\pi}{\lambda} e\sin\theta\right) = d \ \text{(say)}$$

From the method of vector addition, the resultant amplitude is,

$$R = a\frac{\sin nd/2}{\sin d/2} = a\frac{\sin(\pi e\sin\theta/\lambda)}{\sin(\pi e\sin\theta/n\lambda)}$$

$$= a\frac{\sin\alpha}{\sin\alpha/n} \qquad \{\text{Where } \alpha = \pi e \sin\theta/\lambda\}$$

$$= a\frac{\sin\alpha}{\alpha/n} \qquad [\alpha/n \text{ is very small}]$$

$$= na \sin\alpha/\alpha = A\sin\alpha/\alpha \quad \{\text{where } na = A\}$$

Intensity, $I = R^2 = A^2 (\sin\alpha/\alpha)^2$ ------------> (1)

<u>*Principle maxima*</u>: The expression for resultant amplitude R can be written in ascending powers of α as,

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + - - - -\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + - - - -\right]$$

If the –Ve terms vanish, the value of R will be maximum i.e., $\alpha = 0$.

$$\alpha = \pi e \sin\theta / \lambda = 0 \quad \text{or} \quad \sin\theta = 0 \quad \text{or} \quad \theta = 0.$$

The maximum value of R is A and the intensity is proportional to A^2.

Minimum Intensity Position: The intensity will be minimum when $\sin\alpha = 0$. The values of α which satisfy this equation are, $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$, - - - - - -

$$\text{Or} \quad \pi e \sin\theta / \lambda = \pm m\pi \quad \text{or} \quad e \sin\theta = \pm m\lambda \quad \text{where } m = 1,2,3.\ .\ .\ .\ .\text{etc}$$

We obtain minimum intensity on either side of the principal maxima.

Position of secondary maxima

Differentiate Eq (1) w.r.t α and equal to zero

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)\right] = 0 \Rightarrow A^2\,\frac{2\sin\alpha}{\alpha}\,\frac{(\alpha\cos\alpha - \sin\alpha)}{\alpha^2} = 0$$

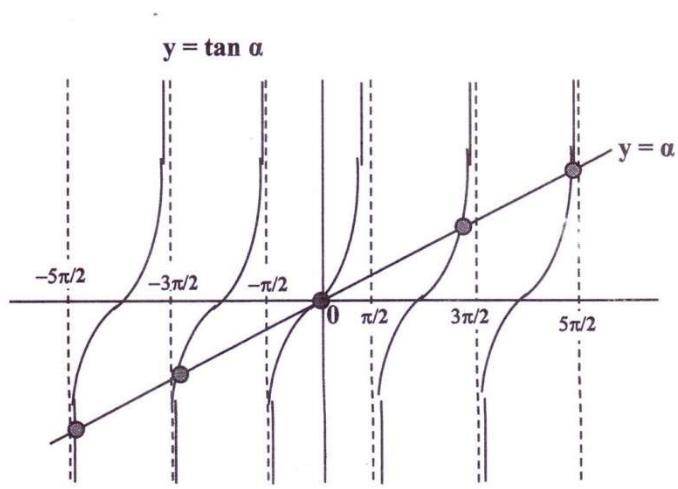
we have ***two solutions*** :

$\sin\alpha = 0$ and

$\alpha\cos\alpha - \sin\alpha = 0$

$\sin\alpha = 0$ gives the value of α for which the intensity is zero. The position of secondary maxima is given by $\alpha\cos\alpha - \sin\alpha = 0 \Rightarrow \alpha\cos\alpha = \sin\alpha$

$$\alpha = \tan\alpha$$

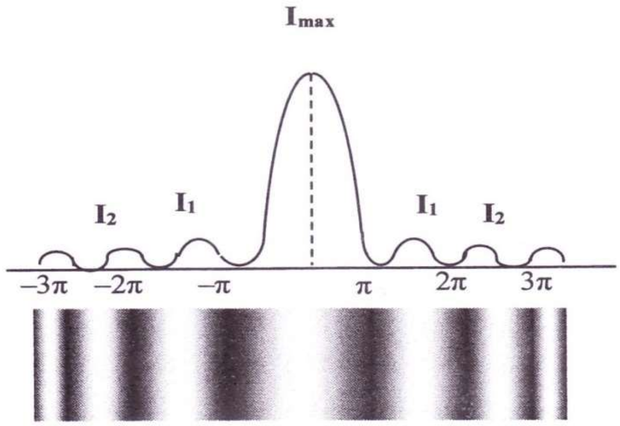


Let us assume that

$y = \alpha$ and $y = \tan\alpha$

The value of α can be obtained graphically by plotting the curve $y = \tan\alpha$ and a straight line $y = \alpha$. The points of intersection of st.line with curves gives the values of α

Intensity distribution graph:-

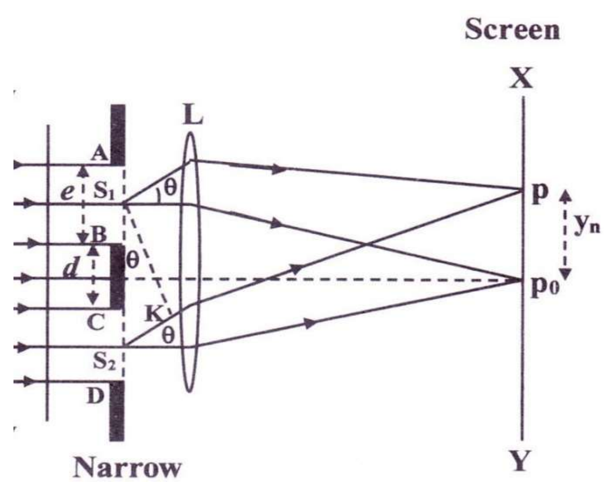


Fraunhofer Diffraction due to double slit

Consider two parallel slits AB and CD each of width 'e' and separated by a distance'd' and perpendicular to plane of the paper. The diffraction pattern produced by two slits is a product of interference pattern produced by two slits separated by a distance (e+d) and the single slit diffraction. L is a collecting lens and XY is a screen perpendicular to the plane of the paper. P is a point on the screen such that OP is perpendicular to the screen. All the secondary waves travelling in a direction parallel to OP come to focus at P. Therefore, P corresponds to the position of the central bright maximum or principal maximum.

Let S1 and S2 be two effective point sources placed at the middle of the two slits AB and CD. Each source (S1 & S2) emits secondary wavelets of amplitude $A\frac{sin\alpha}{\alpha}$ where $\alpha = \frac{\pi e sin\theta}{\lambda}$.

The secondary wavelets travelling at an angle θ with the normal are focused at a point P_1 on the screen. The point P_1 is of the minimum intensity or maximum intensity depending upon the path difference between the secondary waves originating from the corresponding points of the wavefronts of amplitude $A\frac{sin\alpha}{\alpha}$ and a phase difference δ(say).



To calculate δ draw a perpendicular S_1K on S_2K. The path difference between the wavelets from S1 and S2 in the direction θ.

$=S_2K$

$= (e+d)\sin\theta$

$\therefore$ Phase difference $\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta$

The resultant amplitude at P_1 due to the wavelets from S_1 and S_2 can be calculated by using vector addition theorem.

From Figure

$(OH)^2 = (OG)^2+(GH)^2+2(OG)(OH)\cos\delta$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{A\sin\alpha}{\alpha}\right)\cos\delta$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 [1+1+2\cos\delta]$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 [2+2\cos\delta]$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 2[1+\cos\delta]$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 2[1+2\cos^2\frac{\delta}{2}-1]$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 2[2\cos^2\frac{\delta}{2}] \quad \Rightarrow \quad R^2 = 4\left(\frac{A\sin\alpha}{\alpha}\right)^2 \cos^2\frac{\delta}{2}$$

$$R^2 = 4\frac{A^2\sin^2\alpha}{\alpha^2}\cos^2\left[\frac{\pi}{\lambda}(e+d)\sin\theta\right] \Rightarrow R^2 = 4\frac{A^2\sin^2\alpha}{\alpha^2}\cos^2\beta$$

where $\beta = \frac{\pi}{\lambda}(e+d)\sin\theta$ ………………..(2)

The resultant intensity at point P is given by

$$I = R^2 = \frac{4A^2\sin^2\alpha}{\alpha^2}\cos^2\beta \quad ….. \quad (3)$$

Eq (3) shows the resultant intensity distribution is a product of two terms:

(1) The diffraction pattern produced by a single slit of width ‘e’ is represented by the two point sources $A^2(\sin\alpha/\alpha)^2$

(2) $\cos^2\beta$ represents the interference pattern produced by the two point sources separated by a distance e+d

If the slit width is very small then the variation of the term $(\sin\alpha/\alpha)^2$ may be neglected and the resultant distribution is equal to the interference pattern produced in the Young's double slit experiment. i.e., $I = 4A^2$

Position of maxima and minima:

The interference maxima occurs when $\cos\beta = 1$ or $\beta = \pm n\pi = 0, 1\pi, 2\pi, 3\pi, \ldots\ldots$ (or)

$$(e+d)\sin\theta = \pm n\lambda \text{ where } n = 0,1,2,3,\ldots\ldots$$

The position of minima occurs when $\sin\alpha = 0$ $(\alpha \neq 0)$ and $\cos\beta = 0$

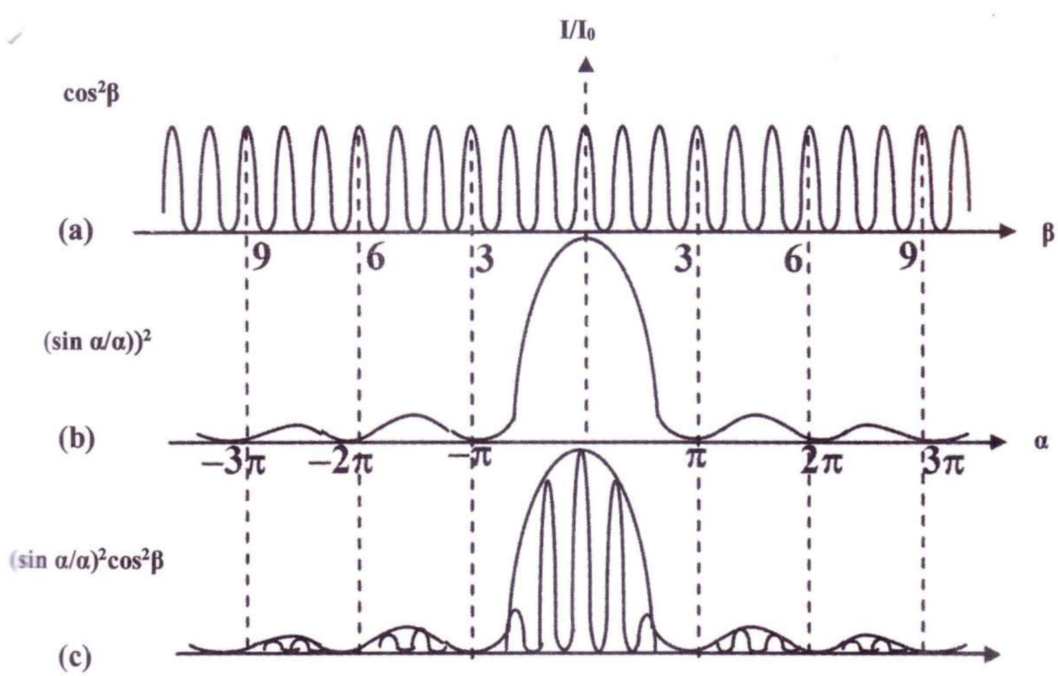
$$\alpha = \pm m\pi = \pi, 2\pi, 3\pi, \ldots\ldots \quad (or) \quad e\sin\theta = \pm m\lambda$$

$$\beta = \pi/2, 3\pi/2, 5\pi/2, \ldots\ldots = (2m+1)\pi/2 \quad (or)\ (e+d)\sin\theta = \pm(2m+1)\lambda/2$$

Position of secondary maxima

The position of secondary maxima occurs when:

$$\alpha = 3\pi/2, 5\pi/2, 7\pi/2 \ldots\ldots = (2n+1)\pi/2 \quad \text{where } n = 1,2,3,\ldots\ldots$$



Intensity distribution of Fraunhofer diffraction by two slits, (a) variation of intensity with α, (b) variation of intensity with β and (c) variation of intensity due to both α and β

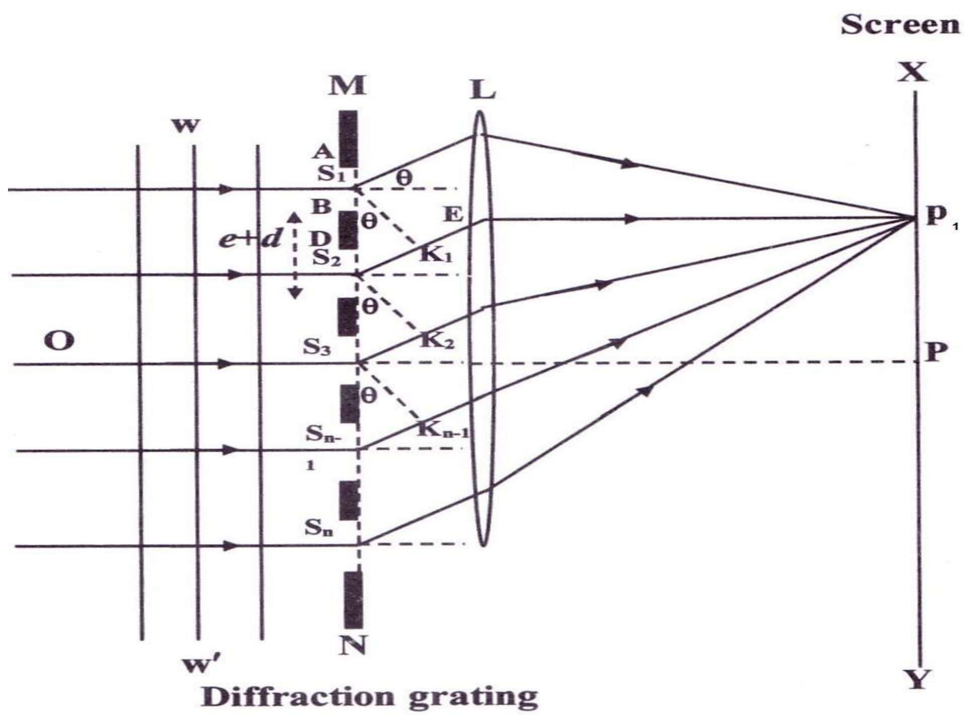
Diffraction grating: An arrangement which essentially consists of a large number of equidistant slits is known as a diffraction grating; the corresponding diffraction pattern is known as the grating spectrum. The grating spectrum provides us with an easily obtainable experimental set up for determination of wavelengths. A good quality grating requires a large number of slits (typically about 30,000 per inch)

Transmission gratings are plane and are ruled on glass surfaces. Refraction gratings may be plane or concave and are ruled on polished metal surfaces. The ruled lines acts as opaque and the spaces between ruled lines acts as slits.

Commercial gratings are produced by taking the cast of an actual grating on a transparent film like cellulose acetate. This transparent film is mounted between two glass plates and are using as a commercial grating.

Plane transmission grating: -

Let a plane transmission grating MN consisting of N slits. Let AB be the slit width 'e' and BD be an opaque portion of width d, then (e+d) is known as grating element. XY is the screen placed perpendicular to the plane of the paper. A parallel beam of light of wavelength λ is incident normally on the grating and each slit sends secondary wavelets in all directions. The secondary wavelets travelling in the same direction of incident light will come to focus at a point P on the screen and will be a central maximum.



Diffraction grating

Secondary waves travelling in a direction inclined at an angle θ with the direction of the incident light. These waves reach point P_1 on passing through the convex lens in different phases. As a result dark and bright bands on both sides of central maximum are obtained. The wavelets processing from all directions in a slit along the direction θ are equivalent to a single wave of amplitude (Asinα/α) starting from the middle point of slit where $\alpha = \pi e \sin\theta / \lambda$.

If the slits are N, then we get N diffracted waves, each from the middle points of the slits. The path difference between two consecutive slits is (e+d) sin θ. Phase difference between two consecutive waves $\delta = \frac{2\pi}{\lambda}$ (e+d) sinθ and is constant, It is 2β

$$\therefore \frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta \text{ ………….. (1)}$$

By the method of vector addition of amplitudes, the direction θ will be

$$R = \frac{A\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta}$$

$$I = R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

The factor $\left(\frac{A\sin\alpha}{\alpha}\right)^2$ gives the distribution of intensity due to single slit while $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ gives the distribution of intensity as combined effects of all the slits.

<u>Intensity distribution</u>

Principle maxima:

The intensity would be maximum when $\sin\beta = 0$

(or) β= ± nπ where n = 0,1,2,3…..

At the same time sin Nβ = 0, so that the factor (sin Nβ/sinβ) becomes indeterminate. Applying the Hospital's rule

$$\lim_{\beta\to\pm n\pi}\frac{\sin N\beta}{\sin\beta} \Rightarrow \lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$\lim_{\beta\to\pm n\pi}\frac{N\cos N\beta}{\cos\beta} = \pm N$$

Hence, $\lim_{\beta\to\pm n\pi}\left(\frac{\sin N\beta}{\sin\beta}\right)^2 = N^2$

The resultant intensity is $\left(\frac{A\sin\alpha}{\alpha}\right)N^2$. The maxima are most intense and are called as principle maxima.

The maximum are obtained for

$\beta = \pm n\pi$ (or) $\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$ (or) (e+d) sin θ = ± nλ

where n = 0,1,2,3…….

For n = 0 corresponds zero order maximum

For n = 1,2……..etc., obtain first, second ..etc., principal maxima

± sing shows that there are two principal maxima of the same order lying on either side of zero order maximum.

For minima:

$$\text{Sin } N\beta = 0 \quad \text{but } \sin\beta \neq 0$$

$$N\beta = \pm m\pi$$

$$N\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

$$N(e+d)\sin\theta = \pm m\lambda$$

where m has all integral values except 0,N,2N, …….nN. Minima are adjacent principal maxima.

Secondary maxima

The intensity of secondary maxima is obtained as follows:

$$\frac{dI}{d\beta} = 0$$

$$\frac{dI}{d\beta} = \left(\frac{A\sin\alpha}{\alpha}\right)^2 .2\left(\frac{\sin N\beta}{\beta}\right)\left[\frac{N\cos N\beta\sin\beta - \sin N\beta\cos\beta}{\sin^2\beta}\right] = 0$$

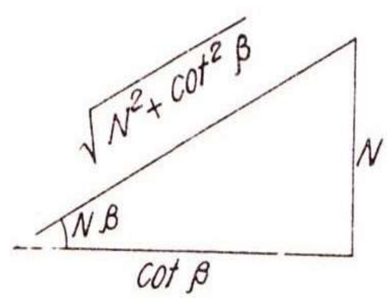
$$N\cos N\beta\sin\beta - \sin N\beta\cos\beta = 0 \Rightarrow N\tan\beta = \tan N\beta$$

The roots of this equation other than those for which β = ± nπ give the positions of secondary maxima.

To find out the value of ($\sin^2 N\beta/\sin^2\beta$) from equations N tan β = tan Nβ

From the fig

$$\sin N\beta = \frac{N}{\sqrt{N^2+\cot^2\beta}}$$



$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{(N^2+\cot^2\beta)X\sin^2\beta}$$

$$= \frac{N^2}{N^2\sin^2\beta+\cos^2\beta}$$

$$= \frac{N^2}{N^2\sin^2\beta+\cos^2\beta+\sin^2\beta-\sin^2\beta}$$

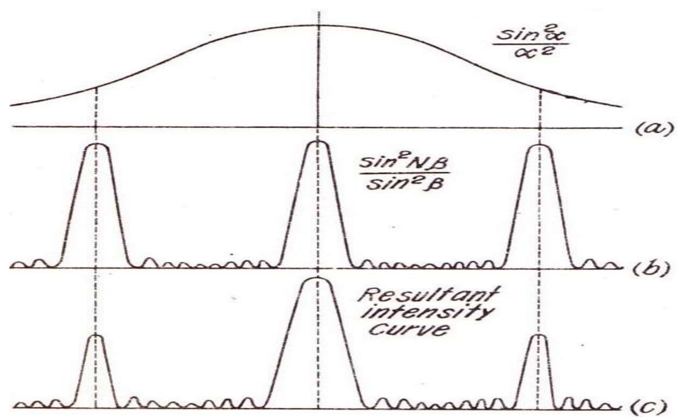
$$= \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

= Intensity of secondary maxima/Intensity of principal maxima

$$= \frac{1}{1+(N^2-1)\sin^2\beta}$$

As the no. of slits increases in grating, relative intensity of secondary maxima with respect to first prinicipal maximum decreases. If N is very large there are no more secondary maxima.

Intensity distribution



Dispersive power and resolving powers of a grating:

Dispersive power: It is defined as the rate of change of angle of diffraction of light with wavelength. Mathematically dispersive power is represented by dθ/dλ.

We know that the condition for primary maxima is,

$$(e + d) \sin\theta = n\lambda \qquad --- \quad (1)$$

where, θ is the angle of diffraction for which n^{th} primary maxima is produced.

Differentiating eq(1) on bothsides, we get

$$(e + d) \cos\theta d\theta = n.d\lambda$$

$$\Longrightarrow \quad d\theta/d\lambda = n / (e + d) \cos\theta \qquad --- \quad (2)$$

Dispersive power = n / (e + d) Cosθ

The value of angular separation is given by,

$$d\theta = n\, d\lambda / (e + d) \cos\theta \qquad --- \quad (3)$$

from eq(3) we can conclude that, the angular seperation

1) depends directly on n (the order of seperation)
2) is inversely proportional to grating element
3) is inversely proportional to Cosθ.

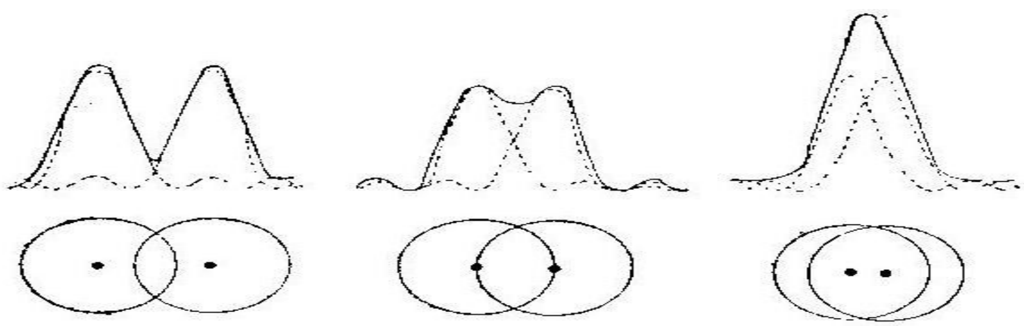
***Resolving power of a grating*:** It is the ability of an instrument to show two very close objects as separate is called its resolving power.

The resolving power of a grating is defined as the ratio of the wavelength of a spectral line to the least difference in wavelength of the next line, which can just be seen as separate.

Rayleigh's criterion for limit of resolution:-

When light from a distant point source and diffracted by a circular opening, is focused by a lens not as a geometrical point but as a disc of finite radius surrounded by one or two faint bright and dark rings are obtained.

If light from two closely situated point sources is focused by a lens then two such like diffraction patterns are produced. The practical criterion for the limit of resolution of two close diffraction patterns was first suggested by Rayleigh and is known as Rayleigh's criterion for resolution. According to this criterion, two point sources are just resolved if the central maximum of the diffraction pattern of one source just falls on the first minimum of the other as shown in fig. In the fig., the intensities of individual patterns are shown by dotted lines but the solid line in each case indicates the resultant intensity of the two patterns.



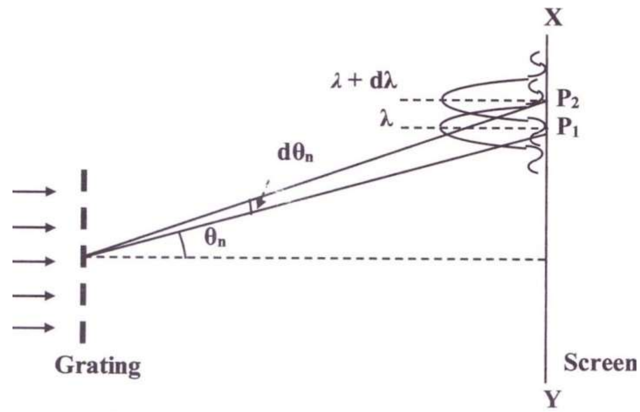
WELL RESOLVED (a) JUST RESOLVED (b) NOT RESOLVED (c)

In fig(a) the two patterns are well resolved because there is a marked decrease in the intensity of light in between the centers of the two patterns. In fig(b), the two patterns are just resolved because the central maximum of one just falls on the 1st minimum of the other. The decrease in intensity in between the centers of the two patterns is just enough to show them as separate. The patterns shown in fig(c) are not resolved because they are two close to each other. The resultant intensity curve is quite smooth without any *dip*.

One of the important properties of a diffraction grating is its ability to separate spectrum lines, which have nearly the same wavelength.

***Explanation*:**

A light source having two wavelengths λ and λ + dλ is normally incident on the grating surface. Let P_1 be n^{th} principal maximum of wavelength λ in the direction θ_n and P_2 be the n^{th} principal maximum of wavelength λ + dλ in the direction $\theta_n + d\theta_n$ shown in fig.



According to Rayleigh criterion these two spectral lines are said to be just resolved when the position of P_2 falls on the first minimum of P_1 and vice versa.

The n^{th} principal maximum of λ in the direction of θ_n is :

$$(e + d)\sin\theta_n = n\lambda \qquad \text{------------ (1)}$$

The first minimum adjacent to n^{th} principal maximum of λ in the direction of $\theta_n + d\theta_n$ is :

$$N(e + d)\sin(\theta_n + d\theta_n) = m\lambda \qquad \text{------------ (2)}$$

The first minimum adjacent to n^{th} principal maximum of λ in the direction of $\theta_n + d\theta_n$ can be obtained by substituting the value m = (nN +1) in Eq.(2)

The first minimum of λ in the direction of $\theta_n + d\theta_n$ is

$$N(e + d)\sin(\theta_n + d\theta_n) = (nN + 1)\lambda \qquad \text{------------------- (3)}$$

The n^{th} principal maximum of λ + dλ in the direction of $\theta_n + d\theta_n$ from eq (1)

$$(e + d)\sin(\theta_n + d\theta_n) = n(\lambda + d\lambda) \qquad \text{-----------------(4)}$$

Multiply eq (4) with N

$$N(e + d)\sin(\theta_n + d\theta_n) = nN(\lambda + d\lambda) \qquad \text{--------------(5)}$$

From eq (3) and (5)

$$(nN+1)\lambda = nN(\lambda + d\lambda)$$

$$\lambda = nNd\lambda$$

$$R = \lambda/d\lambda = nN$$

This equation is called resolving power of grating.

Fresnel's assumptions

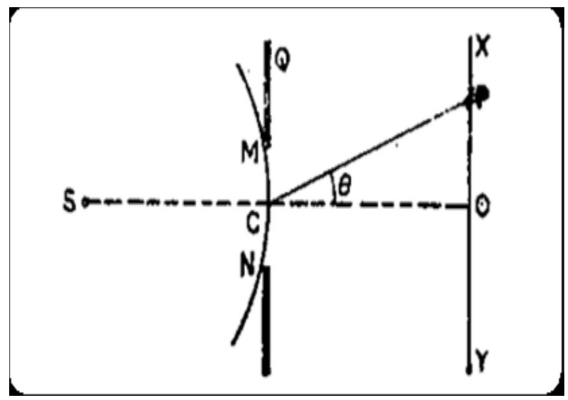
Fresnel in 1815, combined the Huygens principle of wavelet and the principle of interference to explain the bending of light around obstacles and the rectilinear propagation of light.

1. According to Huygens' principle, each point of a wavefront (wavefront is a locus of points in a medium that are vibrating in same phase) is a source of secondary disturbance and wavelets coming from these points spread out in all directions with the speed of light. The envelope of these waves constitutes the next wavelet.

2. According to Fresnel, a wavefront can be divided into many strips or zones called Fresnel zones of small area. The resultant effect at any point will depend on the combined effect of all the secondary waves coming from various zones.

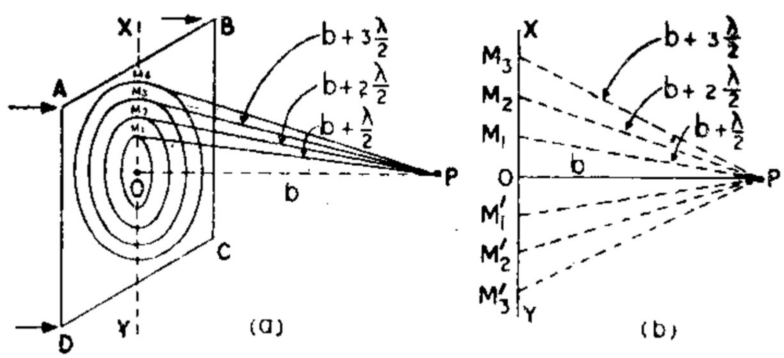
3. The effect at a point due to any zone depends on distance of the point from the zone.

4. The effect will also depend on the obliquity (inclination) of the point with reference to the zone under consideration.



Fresnel's half period zones

ABCD is a plane wave front of monochromatic light of wavelength λ. The diagram shows the plane wavefront as perpendicular to plane of the paper. Consider a point P at a distance b from the wave front at which amplitude due to the wave is to be found. To find the resultant amplitude at P due to entire wavefront, Fresnel assumed the wavefront to be divided into a number of concentric half period zones.



Fresnel's half period zones:

With P as centre and with $M_1P = (b + \lambda/2)$, $M_2P = (b + 2\lambda/2)$,….. as radii, a series of concentric spheres are drawn on the wavefront. These spheres intersect the wavefront in concentric circles. These circles or zones are of radii OM_1, OM_2,….on the wavefront with O as centre.

The secondary waves from any two consecutive zones reach the point P with a path difference of $\lambda/2$ or a time period of $T/2$. Hence these zones are called half period zones. The area of the circle OM_1 is called first half period zone. The area between the circles of OM_2 and OM_1 is called second half period zone and so on. The area between the nth and $(n-1)^{th}$ circle is called the nth half period zone.

To find the radius of a half period zone:

In the diagram, from the right angled triangle OM_1P,

$$OM_1 = \sqrt{M_1P^2 - OP^2} = \sqrt{(b + \frac{\lambda}{2})^2 - b^2}$$

$$OM_1 = \sqrt{(b^2 - 2b\frac{\lambda}{2} + \frac{\lambda^2}{4}) - b^2} \text{ (or) } OM_1 = \sqrt{b\lambda} \text{ (neglecting } \lambda^2/4 \text{ as } b \gg \lambda)$$

$OM_1 = \sqrt{b\lambda}$ is the radius of first half period zone.

The radius of the second half period zone is

$$OM_2 = \sqrt{(M_2P)^2 - (OP)^2} = \sqrt{(b + \frac{2\lambda}{2})^2 - b^2} \text{ Thus } OM_2 = \sqrt{2b\lambda}$$

Similarly, the radius of the nth half period zone is $OMn = \sqrt{(b + \frac{n\lambda}{2})^2 - b^2}$

$$OM_n = \sqrt{nb\lambda}$$

Thus, the radii of 1^{st}, 2^{nd}, …… half period zones are $\sqrt{b\lambda}$, $\sqrt{2b\lambda}$,…. $\sqrt{nb\lambda}$

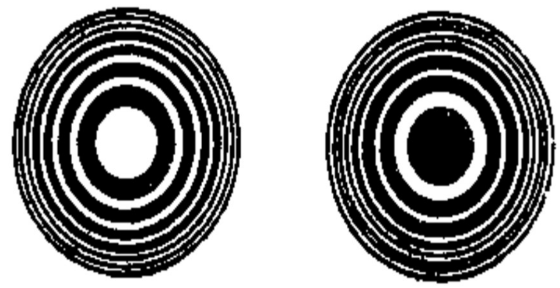
Therefore, the radii of the zones are proportional to the square root of natural numbers.

Zone plate

A zone plate is a specially constructed screen such that light is obstructed from every alternate zone. The correctness of Fresnel's method in dividing a wavefront into half period zones can be verified with the help of zone plate.

A zone plate is constructed by drawing concentric circles on a white paper such that radii are proportional to the square root of the natural numbers. The odd numbered zones (i.e., 1^{st}, 3^{rd}, 5^{th} …) are covered with black ink and a reduced photograph is taken.

The negative of the photograph appears is as shown in Fig. (a). The negative shows odd zones are transparent to incident light and even zones will cut off light. This is a **positive zone plate**. If odd zones are opaque and the even zones are transparent then it is a **negative zone plate**. Fig. (b)



(a) (b)

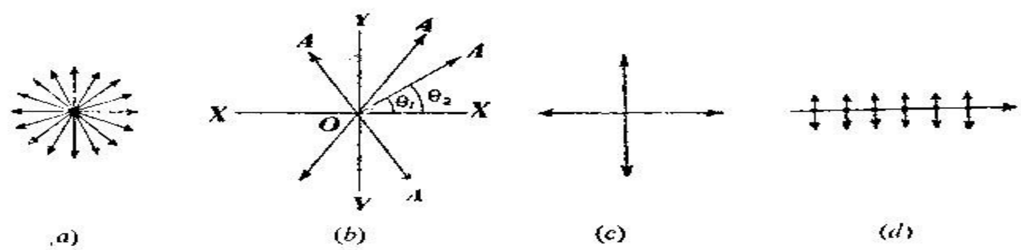
Differences between Zone plate and convex lens

Zone Plate	Convex lens
Focal length of a zone plate is $\frac{1}{f}=\frac{n\lambda}{r_n^2}$	Focal length of lens is $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$
f depends on λ and show chromatic aberration. Forms real image.	f depends on λ and show chromatic aberration. Forms real image.
It has multiple foci. If (2p -1) is the number of half period elements in each zone $f_p=\frac{r_n^2}{(2p-1)n\lambda}$	It has single focus. $\frac{1}{f}=(n-1)(\frac{1}{R_1}-\frac{1}{R_2})$
All the waves reaching the image point through consecutive transparent zones have a path difference of λ.	All waves reaching the image point have same optical path.
f violet > f red	f violet < f red
Intensity of image is less	Intensity of image is greater.

UNIT III - Polarization

Polarisation:

The one-sideness of light is called polarisation. The light, which has acquired the property of one-sideness, is called polarised light. Natural or un polarised light consists of vibrations of electric vector taking place in all transverse directions with random orientation. Such a light may be called symmetrical light. The plane in which vibrations take place is known as the plane of vibration. The plane, which is $\perp_r$ to plane of vibration is known as the plane of polarisation. Light waves are transverse waves. Ordinary eye cannot detect the polarisation. We can represent unpolarised light as follows.

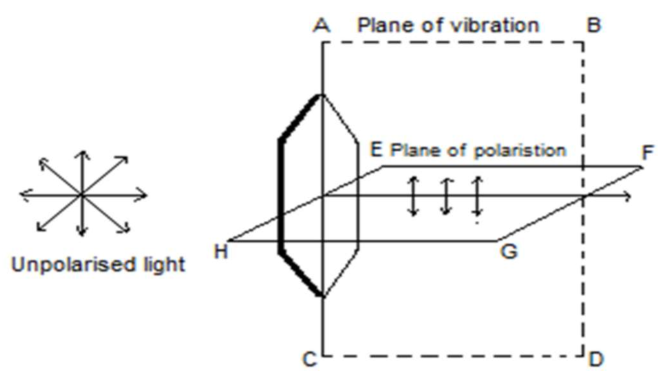


The amplitudes of the electric vectors can be resolved into two mutually $\perp_r$ directions. They may be taken as,

x- component : $A \cos\theta_1 + A \cos\theta_2 + A \cos\theta_3 +$ -------

y- component : $A \sin\theta_1 + A \sin\theta_2 + A \sin\theta_3 +$ -------

i.e., Natural light consists of two kinds of vibrations only at right angles to each other. One in the plane of the paper (double headed arrows) and the other $\perp_r$ to the plane of the paper (dots).



If A is the amplitude of the unpolarised light, A_m is the amplitude of the plane polarised light, then intensity of the plane polarised light is given by, $I = A^2 \cos^2\theta = I_m \cos^2\theta$ -- **Malus law**.

Plane polarised light: When the amplitude of the electric vector changes but the orientation remains constant, then the path traced by the electric vector is a straight line and the light is said to be plane polarised.

Circularly polarised light: When the amplitude of the electric vector of light remains constant but the orientation changes, then the path traced by the electric vector is a circle and the light is said to be circularly polarised.

Elliptically polarised light: When the amplitude of the electric vector and the orientation are changing, then the path traced by the electric vector is a ellipse and the light is said to be elliptically polarised.

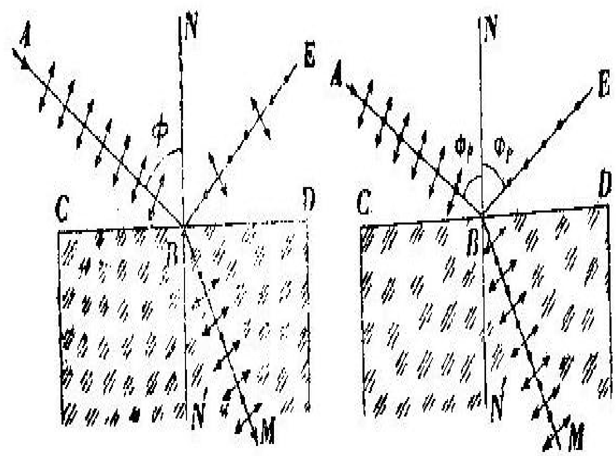
Plane polarised light can be produced by the following three methods

i) By reflection

(ii) By refraction

(iii) By double refraction

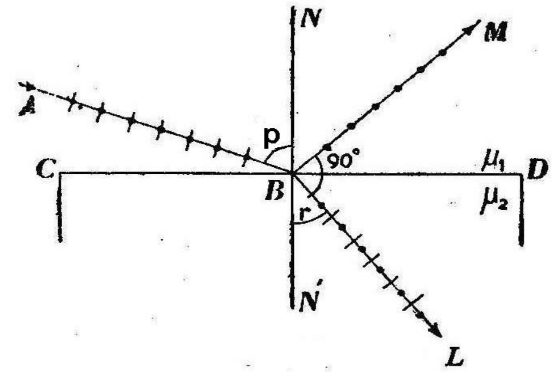
Polarization by reflection: - The simplest way of producing a plane polarised light is by reflection. Malus discovered that when ordinary light is reflected from the surface of a transparent medium like glass or water it becomes partially polarised. The degree of polarisation changes with angle of incidence. At a particular angle of incidence, the reflected light has the greatest percentage of polarised light. This angle of incidence is known as angle of polarisation.



Let a beam of unpolarised light incident on a glass surface CD as shown in figure. Incident beam consists of two vibrations at right angles to each other (i)Vibrations that are $\perp_r$ to the plane of incidence (dot components) (ii) Vibrations Parallel to the plane of incidence (arrow components). For all angles of the incidence both reflected and refracted rays are partially polarised. But when the light is incident at the polarising angle, then none of the arrow component is reflected. So the reflected ray contains only dot components i.e., the reflected ray is completely plane- polarised. The refracted ray is a mixture of arrow and dot components.

Brewster's Law: The tangent of the angle of polarisation (p) is numerically equal to the refractive index (μ) of the medium. i.e., $\mu = \tan p$

He also proved that the reflected and refracted rays are $\perp_r$ to each other.



Let a beam of unpolarised light be incident on a glass surface at polarising angle **p** as shown in fig. (The polarising angle for air – glass is 57^0). A part of the incident light is reflected while a part is refracted. Let **r** be the angle of refraction.

From Brewster's law, $\mu = \tan p$ -- (1)

From Snell'e law, $\mu = \sin p / \sin r$ -- (2)

From (1) and (2)

$$\tan p = \sin p / \sin r \implies \sin p / \cos p = \sin p / \sin r$$

$$\sin r = \cos p = \sin (90 - p) \implies r = 90 - p \quad \text{or} \quad r + p = 90^0$$

∴ The reflected and refracted rays are at right angles to each other.

Malus Law

Malus' law states that the intensity of plane-polarized light that passes through an analyzer varies as the square of the cosine of the angle between the plane of the polarizer and the transmission axes of the analyzer. The law helps us quantitatively verify the nature of polarized light.

Point 1 – When Unpolarized light is incident on an ideal polarizer the intensity of the transmitted light is exactly half that of the incident unpolarized light no matter how the polarizing axis is oriented.

Point 2 – An ideal polarizing filter passes 100% of incident unpolarized light, which is polarized in the direction of the filter's (Polarizer) Polarizing axis.

From point (1) and point (2) we can assume $I = I_0 \cos^2 \phi$

The average value of I (< I >):

We know

$$\langle I \rangle = \langle I_0 \rangle \langle \cos^2 \phi \rangle$$

$$\langle \cos^2 \phi \rangle = \frac{1}{2}$$

Which satisfies point (2) mentioned above.

To show point (1), let us consider $\phi = 0$

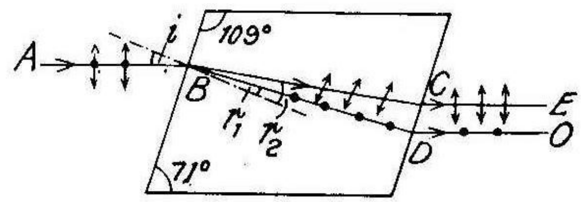
That implies $\cos^2\phi = 1$

$I = I_0$

Double refraction:

When a ray of light is refracted by a crystal of calcite it gives two refracted rays. This phenomenon is called double refraction.

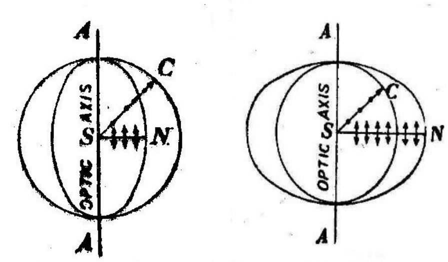
When a ray of light is incident on the calcite crystal, it is refracted along two paths inside the crystal i) along BC (angle of refraction r_2) and ii) along BD (angle of refraction r_1). One ray obeys Snell's law and it is called ordinary ray (O- ray) the other ray which doesn't obey Snell's law is called extra ordinary ray (E- ray). These two rays emerge out along DO and CE, which are parallel. Velocity of O- ray is the same as that of the incident ray, while the velocity of E- ray depends on its direction of travel thru the crystal.



According to Huygen's, from the disturbance of the particels two types of wave fronts are produced. For the O- ray, the velocity of light is the same in all directions and the wavefront is spherical. And for E- ray, the velocity varies with the direction and the wavefront is an ellipsoid. There are two points where these two wavefronts touch each other. The line joining these two points is the optic axis. Based on the refractive index of E- ray (μ_e) there are two kinds of uniaxial crystals: +ve crystals and –ve crystals.

In +ve crystals the elliptical wavefront is surrounded with circular wavefront. In this $\mu_e > \mu_o$.
Ex: ice, Quartz etc.,

In –ve crystals the spherical wavefront is surrounded with elliptical wavefront. In this $\mu_e < \mu_o$.
Ex: Calcite, Tourmaline etc.,



Quarter - wave plate: It is a uniaxial doubly refracting crystal plate, cut with its axis parallel to the refracting faces, and can produce a phase difference of $\pi/2$ or a path difference of $\lambda/4$ between the ordinary and extra-ordinary rays.

When a plane polarised light falls normally on a crystal of this type, it splits up into ordinary and extra ordinary plane polarised lights. They travel along the same paths but with different velocities.

The velocity of E- ray is greater than the velocity of O-ray. As a result a phase difference is introduced between them.

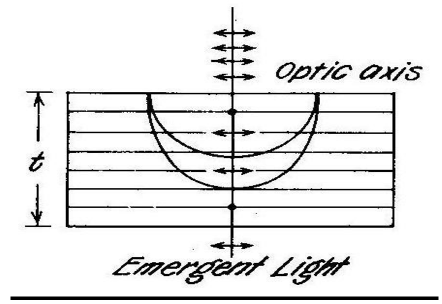
Let t be the thickness of the plate. If μ_0 and μ_e be the refractive indices of the crystal for O- ray and E- ray, then the path difference between O- ray and E- ray is $(\mu_0 - \mu_e)t$ ($\mu_0 > \mu_e$)

For a quarter wave plate this distance should be $\lambda/4$

$$(\mu_0 - \mu_e)t = \lambda/4$$

or $$t = \lambda/ 4(\mu_0 - \mu_e)$$

For positive crystal, $t = \lambda/ 4(\mu_e - \mu_0)$



Half wave plate:

doubly refracting crystal plate, cut with its axis parallel to the refracting faces, and can produce a phase difference of π or a path difference of $\lambda/2$ between the ordinary and extra-ordinary rays.

When a plane polarised light falls normally on a crystal of this type, it splits up into ordinary and extra ordinary plane polarised lights. They travel along the same paths but with different velocities. The velocity of E- ray is greater than the velocity of O-ray. As a result a phase difference is introduced between them.

Let t be the thickness of the plate. If μ_0 and μ_e be the refractive indices of the crystal for O- ray and E- ray, then the path difference between O- ray and E- ray is $(\mu_0 - \mu_e)t$ ($\mu_0 > \mu_e$)

For a quarter wave plate this distance should be $\lambda/2$

$$(\mu_0 - \mu_e)t = \lambda/2$$

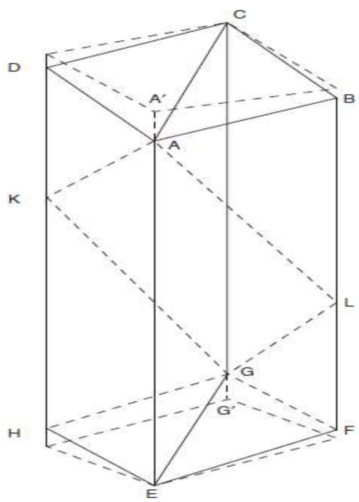
or $$t = \lambda/ 2(\mu_0 - \mu_e)$$

For positive crystal, $t = \lambda/ 2(\mu_e - \mu_0)$

Nicol's Prism:

Principle: When an ordinary light is transmitted through a calcite crystal, it splits into O- ray and E- ray which are completely plane polarised with vibrations in two mutually $\perp_r$ directions. If one ray is eliminated by some means, then the emergent beam will be plane polarised.

Construction: A calcite crystal whose length is three times as its width is taken. The end faces of this crystal are grounded in such a way that the angle in the principal section becomes 68^0 and 112^0 instead of 71^0 and 109^0. And the crystal is cut into two pieces by a plane $\perp_r$ to the principal section as well as the end faces PR and QS. The two cut surfaces are grounded and polished optically flat and then cemented together with Canada balsam. The refractive index of Canada balsam lies between the refractive indices for the O- ray and E- rays for calcite. For Sodium D lines, the values are $\mu_{0\text{- ray}} = 1.658$, $\mu_{e\text{- ray}} = 1.486$ and $\mu_{\text{Canada balsam}} = 1.55$



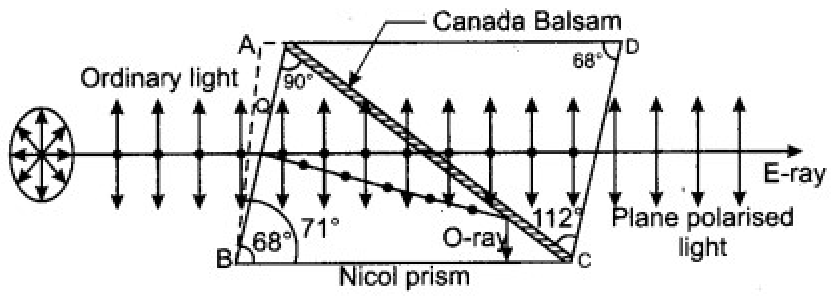


Fig. 12.36

Working: when a beam of light AB enters the faces PR in direction Parallel to the long side, it is doubly refracted into two plane polarised beams BO (O– ray) and BE (E- ray). Since $\mu_{\text{Canada balsam}} < \mu_0$, Canada balsam acts as a rarer medium for O- ray and denser medium for E- ray. If the angle of incidence of O- ray at the calcite-balsam surface becomes greater than the corresponding critical angle (69^0) then the O- ray is completely reflected at calcite-balsam surface and is absorbed by the tube containing the Nicol's prism. Now, critical angle of O-ray with respect to Canada balsam is

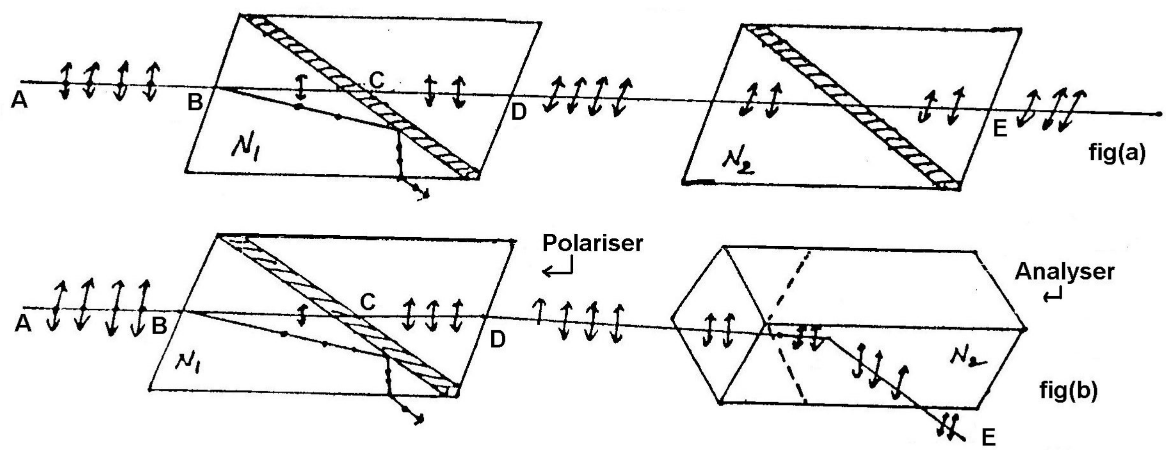
$$Sin\ c = 1.658/1.550$$

Which implies that ; $C = Sin^{-1}(1.550/1.658) = 69°$ $\quad \therefore C = 69°$

The angle of incidence on Canada Balsam depends upon the angle which A'B makes with blunt edge BC' and also on breadth of length ratio of the crystal. This was the only reason that length of crystal is chosen thrice of breadth and natural angle 71° is reduced to 68°. Because by doing so, 0-ray falls on Canada Balsam layer at an angle greater than critical angle *C* so it is totally internally reflected and absorbed, whereas E-ray is transmitted. The transmitted extraordinary ray is plane polarised having vibrations parallel to principal section of the Nicol prism. Thus, Nicol prism act as a polariser

Note: If the angle of incidence on the face PR is greater than 14^0, then the O- ray will strike the balsam layer at an angle less than the critical angle and so will be transmitted.

Uses: Nicol's prism can be used both as polariser and analyser. When two Nicol's are arranged co-axially as shown in fig., the first Nicol which produces plane polarised light is known as polariser while the second which analyses the polarised light is known as analyser. When the two Nicol's are placed with their principal sections parallel to each other as shown in fig(a), then the E- ray transmitted by one is freely transmitted by the other. If the second prism is gradually rotated, then the intensity of E- ray gradually decreases and when the two Nicol's are at right angles to each other [fig(b)] no light comes from second prism.



Production and detection of elliptically and circularly polarised light:

When a plane-polarised light is incident normally on a Quarter wave plate on entering the crystal, it splits up into two components, O-ray and E-ray. Both the rays travel along the same direction but with different velocities. When the rays have travel through the thickness d of the crystal, a phase difference of δ is introduced between them.

Theory: Let the amplitude of the incident plane polarised light on the crystal is A and it makes an angle θ with the optic axis. Therefore, the amplitude of the O- ray vibrating along PO is A Sinθ and the amplitude of the E- ray vibrating along PE is A Cosθ. Since a phase difference δ is introduced between the two rays after passing through a thickness d of a crystal, the rays after coming out of the crystal can be represented in terms of two simple harmonic motions at right angles to each other.

For the E- ray, $x = A \cos\theta . \sin(\omega t + \delta)$ --- (1)

For the O- ray, $y = A \sin\theta . \sin \omega t$ --- (2)

Substitute $A \cos\theta = a$ and $A \sin\theta = b$ in eq's (1) and (2) we get,

$x = a \sin(\omega t + \delta)$ --- (3)

$$y = b \sin \omega t \qquad \text{--- (4)}$$

From eq(3), $x/a = \sin \omega t \cos \delta + \cos \omega t \sin \delta$ --- (5)

From eq(4), $y/b = \sin \omega t$ and $\cos \omega t = 1 - y^2/b^2$ --- (6)

Substitute eq(6) in (5), $\frac{x}{a} = \frac{y}{b} \cos \delta + 1 - \frac{y^2}{b^2} . \sin \delta$

$$\frac{x}{a} - \frac{y}{b} \cos \delta = 1 - \frac{y^2}{b^2} . \sin \delta$$

Squaring and rearranging

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos \delta = \sin^2 \delta \qquad \text{--- (7)}$$

This is the general equation of an ellipse.

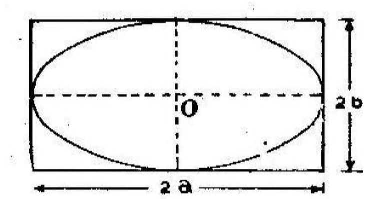
Special Cases.

Case(1) :-

When $\delta = \pi / 2$, $\cos \delta = 0$, $\sin \delta = 1$

From eq(7), $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

This represent the equation of a symmetrical ellipse. The emergent light in this case will be **elliptically polarised** if a = b. The vibrations of the incident plane-polarised light on the crystal should not make an angle of 45^0 with the direction of the optic axis.

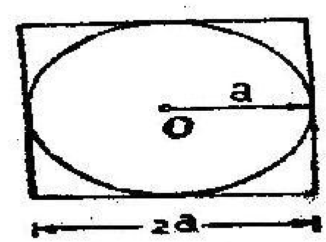


Case (2):-

When $\delta = \pi / 2$ and $a = b$

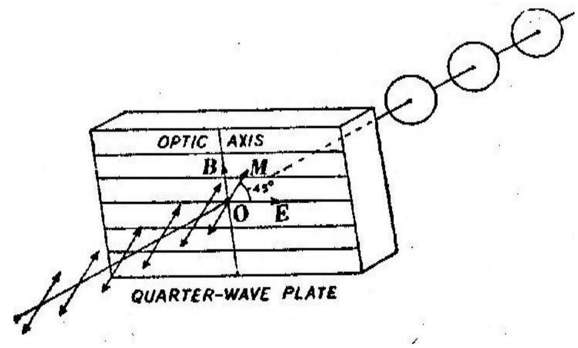
From eq(7), $x^2 + y^2 = a^2$

This represents the equation of a circle of radius a. The emergent light will be **circularly polarised.** Here the vibrations of the incident plane-polarised light on the crystal make an angle of 45^0 with the direction of the optic axis.

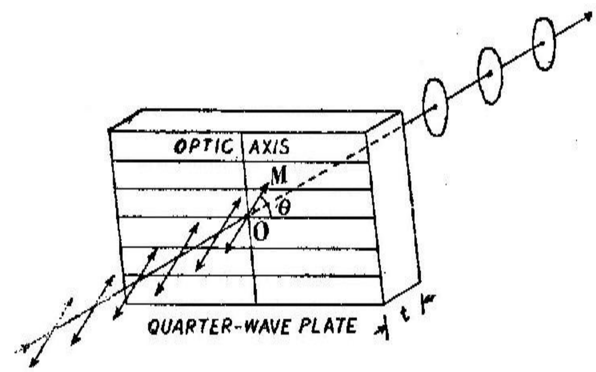


<u>Detection of Circular and elliptical polarized light :-</u>

Circular polarized light – The original sample is passed through a quarter wave plate and the emergent light is analysed by a Nicol which is rotating about the emergent light as the axis. If the intensity varies from zero to maximum the given sample is circularly polarized and at the same time the original sample intensity should not be changed when we analysed by a rotating Nicol



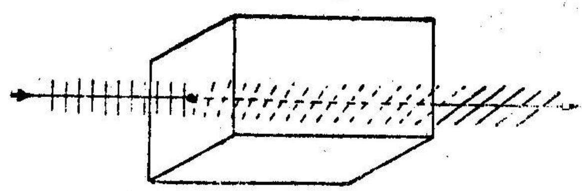
Elliptical polarized light - The original sample is passed through a quarter wave plate and the emergent light is analysed by a Nicol which is rotating about the emergent light as the axis. If the intensity varies from zero to maximum the given sample is circularly polarized and at the same time the original sample intensity should be changed but never completely extinguished when we analysed by a rotating Nicol.



Optical activity: Certain substances like quartz, sugar crystal etc., rotate the plane of vibration of a plane polarised light passing through them. In fig. a quartz crystal on which plane polarised light is incident with its plane of vibration parallel to its optic axis is shown. When the plane polarised light travels inside the crystal its plane of vibration is slowly rotated about the direction of propagation. In the emergent light, the vibrations are in some other plane. This rotation of plane of vibration can be explained with the following experiment. In fig(a) two Nicol's are set in a crossed position. The field of view is dark. Now a quartz plate cut with its faces parallel to the optic axis is introduced in between two Nicol's[fig(b)], the field of view is not dark. But by slightly rotating the second Nicol N_2 the field of view is again dark. The light emerging from quartz is still plane polarised but the plane of polarisation has rotated through certain angle.

The property of rotating the plane of vibration of plane polarised light about its direction of travel by some crystal is known as optical activity.

Those substances like quartz, sugar in solution, cinnabar etc., which rotate the plane of vibration (and also of polarisation) are known as optical active substances.



There are two types of optical active substances. The substances which rotate the plane of polarisation in clock wise (when looking against the direction of light) are called as **dextro-rotatory** or right handed [ex:- Quartz crystal, cane sugar].

The substances which rotate the plane of polarisation in the anti-clock wise direction are called as **leavo-rotatory** or left handed [ex:- fruit sugar].

Specific rotation:

The specific rotation of a substance at a particular temperature and for a given wavelength of light is defined as the rotation produced to the plane of polarisation of a plane polarised light by a decimetre (10cm) length of liquid when the concentration of active substance is 1 gram per cc of solution.

$$\theta s = \frac{\theta}{l X c} = \frac{\text{degrees}}{dm X g.cm^{-3}}$$

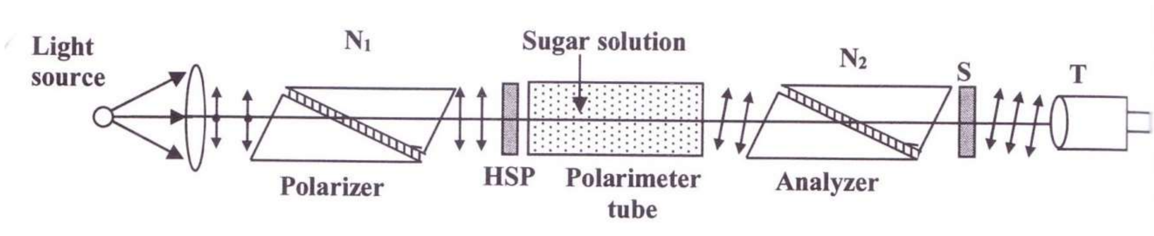
Polarimeter:

A polarimeter is an optical instrument used for the determination of the optical activity of certain solids, liquids, and solutions by measuring the angle of rotation of polarized light as it passes through them. The measured rotation angle can be used to calculate the concentration of sugar solution, peptides, and volatile oils.

Laurent's Half Shade Polarimeter (Saccharimeter)

Principle: *When the polarizer is used to find the optical rotation of sugar solution, then such a polarimeter is known as Saccharimeter. If the specific rotation of the sugar is known, the concentration of the sugar in the solution can be estimated.*

Construction: It consists of two Nicol prisms N_1 and N_2, N_1 acts as polarizer while N_2 as analyzer. Both are placed at some distance apart. N_2 is capable of rotation with respect to the common axis of N_1 and N_2. There is an half shade plate (HSP) having half-wave plate of quartz Q which covers one-half of the field of view of the telescope while the other half G a glass plate which absorbs the same amount of light as the quartz plate and is placed after N_1. An optically active substance (sugar solution) is filled in the tube and closed with cover slips and metal covers and introduce between N_1 and N_2. A telescope is placed after N_2 and is focussed on HSP.



Working: A monochromatic light is incident on the lens L which renders into parallel beam. The unpolarized light after passing through the Nicol prism N1 becomes plane polarized with its vibrations in the principal plane of N1. The plane polarized light now passes through the HSP and then through polarimeter tube T which contains the optically active solution. The emergent light from tube on passing through the analyzer N2 can be viewed through the telescope.

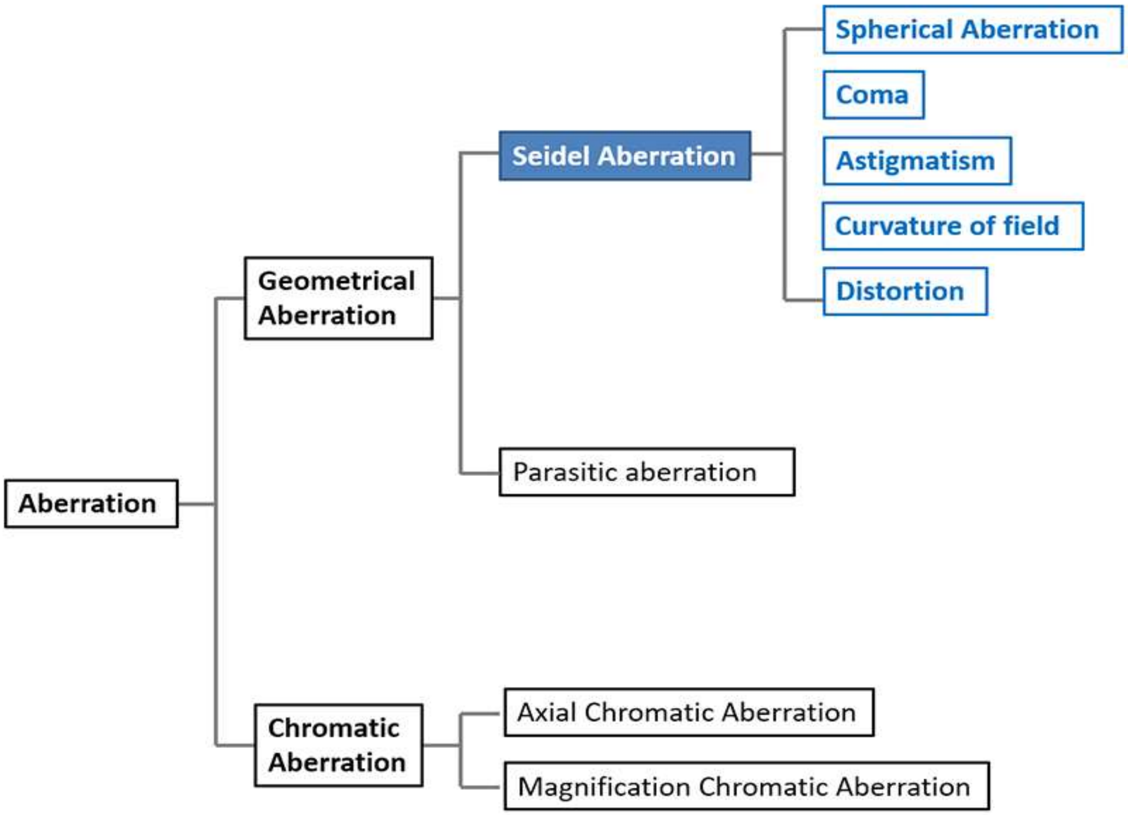
The tube is filled with pure water without bubbles and placed in tis position. Now, focus the telescope on the HSP and the analyzer is rotated till the field of view of the telescope is bright i.e., both the halves HSP should be equally bright. The position of the analyzer is noted on the circular scale S. The tube is filled with sugar solution of known concentration without bubbles and placed it in the same position. The analyzer is now adjusted until the field of view of the telescope is again equally bright. The new position of the analyzer is noted on the circular scale. The difference of the two readings will give the angle of rotationθ. From the known values of *'l'* and '*c* ', the specific rotationθ, is calculated using the relation:

$$\theta s = \frac{\theta}{lXc}$$

Now, the tube is filled with sugar solution of unknown concentration without bubbles and placed in the same position. The analyzer is now again adjusted until the field of view of the telescope is again equally bright. The new position of the analyzer is again noted on the circular scale. By knowing values of θ and *l*, the concentration of sugar solution can be calculated using the following formula:

$$c = \frac{\theta}{lX\theta s}$$

UNIT IV – ABBERRATIONS



Aberration: The deviations from the actual size shape and positions of an image as calculated by simple equations are called chromatic aberrations

The defects are mainly

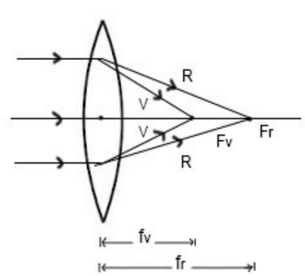
Due to of light: If the light is not monochromatic as (WHITE) then the image becomes multicoloured and the defect is known as ***chromatic aberration***.

Due to optical System: Even with monochromatic light several defects in shape of image are found. They are **five** types

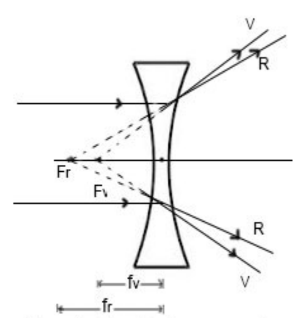
(a) Spherical Aberration (b) Coma (c) Astigmatism (d) Distortion (e) Curvature

Chromatic aberration

The image of white object formed by a lens is coloured and blurred. This defect of image is known as chromatic aberration. The chromatic aberration is of **two** types.



Chromatic aberration in a convex lens.

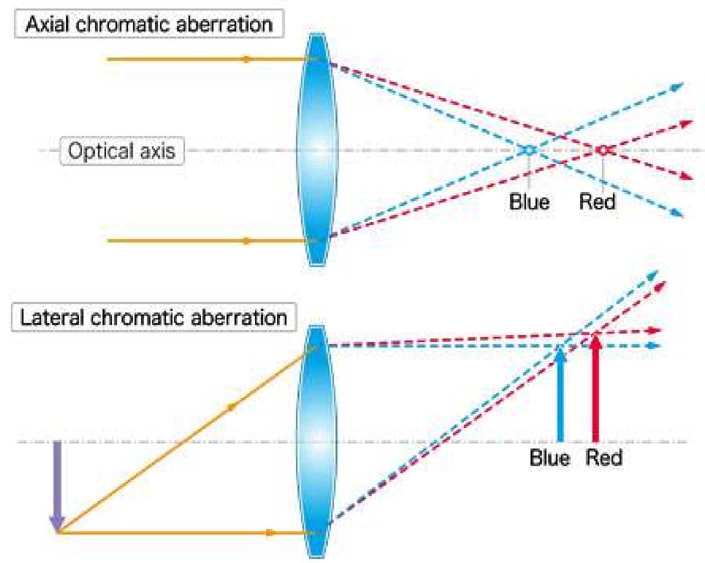


Chromatic aberration in a concave lens.

(i) Longitudinal Chromatic aberration (or) Axial chromatic aberration

(ii) Lateral chromatic aberration (or) Transverse chromatic aberration

Longitudinal Chromatic aberration



The formation of images of different colours on different positions along the axis as knows a longitudinal or axial chromatic aberration. The difference between I_V and I_R is a measure of longitudinal chromatic aberration

∴ longitudinal chromatic aberration = $I_R - I_V$

Lateral chromatic aberration

The images of different colours are formed of different sizes. This defect is known as Lateral chromatic aberration.

$$magnification = \frac{Size\ of\ the\ image}{size\ of\ the\ object}$$

$$= \frac{Distance\ of\ the\ image}{Distance\ of\ the\ object}$$

Calculation of Longitudinal chromatic aberration of a thin lens:

Case (i) When the object is at infinite distance

Case (ii) When the object is finite distance

When the object is situated at infinity:

The focal length f of a lens is given by $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$

If f_V, f_R and f_Y be the focal lengths of the lens for violet, Red and Yellow (mean) coloures and μ_V, μ_R and μ_Y are the refractive indices respectively.

$$\frac{1}{f_V} = (\mu_V - 1)(\frac{1}{R_1} - \frac{1}{R_2}) \text{---------------}(1)$$

$$\frac{1}{f_R} = (\mu_R - 1)(\frac{1}{R_1} - \frac{1}{R_2}) \text{---------------}(2)$$

$$\frac{1}{f_Y} = (\mu_Y - 1)(\frac{1}{R_1} - \frac{1}{R_2}) \text{---------------}(3)$$

Subtracting Equation from (2) from equation (1)

$$\frac{1}{f_V} - \frac{1}{f_R} = (\mu_V - \mu_R)(\frac{1}{R_1} - \frac{1}{R_2}) \quad \Rightarrow \quad \frac{f_R - f_V}{f_V f_R} = (\frac{\mu_V - \mu_R}{\mu_Y - 1})(\mu_Y - 1)(\frac{1}{R_1} - \frac{1}{R_1})$$

$$\frac{f_R - f_V}{f_Y^2} = \frac{(\mu_V - \mu_R)}{(\mu_Y - 1)}(\frac{1}{f_Y}) \qquad (\because f_V f_R = f_Y^2)$$

$$\frac{f_R - f_V}{f_Y^2} = \omega(\frac{1}{f_Y}) \left[\because \omega = \frac{(\mu_V - \mu_R)}{(\mu_Y - 1)}\right]$$

$$\boxed{f_R - f_V = \omega f_Y} \text{------------}(4)$$

$f_R - f_V =$ Longitudinal chromatic aberration

The Longitudinal chromatic aberration for a thin lens for and object at infinity is equal to the product of its mean focal length and dispersive power of its lens.

(ii) **The object is situated at a finite distance**

For a thin lens the relation between u, v and f is given by

$$\frac{1}{u} - \frac{1}{v} = \frac{1}{f} \qquad \text{-----------------}(5)$$

When the object is placed at a fixed finite distance of image and focal length vary with colour. Hence, differentiating eq (5), we have

$$-\frac{dv}{v^2} = -\frac{df}{f^2} \qquad \Rightarrow \qquad \frac{dv}{v^2} = \frac{df}{f^2} \text{-------------}(6)$$

If the image distances wan focal lengths for violet and red colours be v_Y, v_R and f_V, f_R respectively, then

$dv = v_R - v_V$ and $df = f_R - f_V = \omega f_Y$ {from eq (4)} -------------(7)

Substituting the values of dv and df from eq(7) in eq (6) we have

$$\frac{v_R - v_V}{v_Y^2} = \frac{\omega f_Y}{f_y^2} = \frac{\omega}{f_Y} \qquad \{\because v = v_y \text{ and } f = f_Y \text{ (for yellow colour)}\}$$

$$v_R - v_V = \frac{\omega}{f_Y} v_y^2$$ ----------------(8)

Longitudinal chromatic aberration depends upon

(a) dispersive power of the lens material

(b) mean focal length f_Y

(c) the image distance for mean ray v_Y which depends upon *u,* the object distance

ACHROMATISM OF LENSES:

When the two lenses are placed in such a way that the image formed by them is free from chromatic aberration, the combination of lenses is called as *chromatic combination* and the phenomenon is known as *achromatism*. ***The minimization or removal of chromatic aberration is known as achromatism***.

It is possible by a combination of a suitable convex lens with a concave lens can free from chromatic aberration. Such combination is known as achromatic doublet. To achieve achromatic doublet a crown glass convex lens of low focal length (i.e., high power) and flint glass concave lens of greater focal length (i.e., low power) are used. The conditions of achromatism in two important cases:

(a) **When two lenses are in contact with each other** (used in microscopes and telescopes as objectives)

(b) **When two lenses are separated by a distance** (used for the eyepieces of optical instruments)

Case (1) Achromatism for two lenses in contact:

Achromatic doublet is formed in such a way that all colors come to focus at one point, i.e., all colors have the same focal length. i.e., focal length of chromatic doublet is independent of the refractive index μ.

For a single lens $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$ ---------------(1)

The change in focal lengths corresponding to a change in refractive index can be obtained by differentiating equation (1)

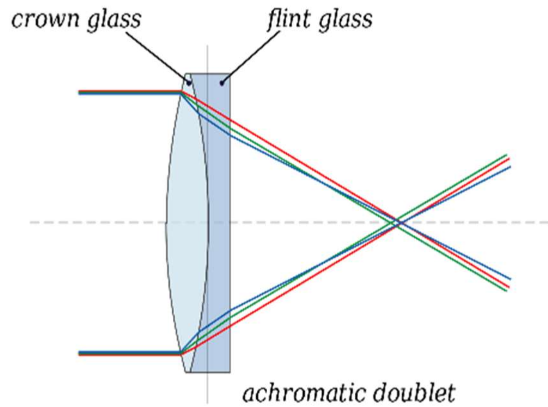
$$d(\frac{1}{f}) = d\mu(\frac{1}{R_1} - \frac{1}{R_2})$$ --------------(2)

Dividing equation (2) by equation (1)

$$fd(\frac{1}{f}) = \frac{d\mu}{\mu - 1} \quad \Rightarrow \quad d(\frac{1}{f}) = \frac{d\mu}{\mu - 1}(\frac{1}{f}) = \frac{\omega}{f}$$ --------------(3)

Let f_1 and f_2 be the focal lengths of the two lenses in contact and ω_1 and ω_2 are dispersive powers of lenses respectively. If F be the focal length of the combination, then

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2} \text{------------(4)}$$



Change in (1/F) can be obtained by differentiating Eq.(4)

$$d(\frac{1}{F})=d(\frac{1}{f_1})+d(\frac{1}{f_2})$$

From Eq.(3) $d(\frac{1}{f_1})=\frac{\omega_1}{f_1}$; $\quad d(\frac{1}{f_2})=\frac{\omega_2}{f_2}$

$$d(\frac{1}{F})=\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2} \text{--------------(5)}$$

For achromatic combination F and hence (1/F) should not change with colour. i.e., d(1/F) = 0

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$$

The ratio of the focal lengths of the two lenses in an achromatic doublet is given by

$$\boxed{\frac{f_1}{f_2}=-\frac{\omega_1}{\omega_2}}$$

This is required condition.

Case (ii) Achromatism for two lenses seperated by a distance

Consider two convex lenses of focal lengths f_1 and f_2 separated by a suitable distance x. The material of the two lenses is the same hence their dispersive power is same. If F is the combined focal length of two lenses, then

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{x}{f_1 f_2} \text{----------(1)}$$

The change in $\frac{1}{F}$ as the refractive index changes can be obtained by diff. Eq.(1)

$$d(\frac{1}{F})=d(\frac{1}{f_1})+d(\frac{1}{f_2})-d(\frac{x}{f_1 f_2})$$

$$d(\frac{1}{F})=d(\frac{1}{f_1})+d(\frac{1}{f_2})-\frac{x}{f_2}d(\frac{1}{f_1})-\frac{x}{f_1}d(\frac{1}{f_2}) \text{---------------(2)}$$

We know that $d(\frac{1}{f_1}) = \frac{\omega_1}{f_1}$; $\qquad d(\frac{1}{f_2}) = \frac{\omega_2}{f_2}$

$$d(\frac{1}{F}) = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{x}{f_2}\frac{\omega_1}{f_1} - \frac{x}{f_1}\frac{\omega_2}{f_2} \text{-------------(3)}$$

For an achromatic combination the focal length F or $(\frac{1}{F})$ should not change with colour. i.e., $d(\frac{1}{F})=0$

$$0 = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{x}{f_2}\frac{\omega_1}{f_1} - \frac{x}{f_1}\frac{\omega_2}{f_2}$$

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \frac{x}{f_2}\frac{\omega_1}{f_1} + \frac{x}{f_1}\frac{\omega_2}{f_2} \qquad \Rightarrow \qquad \frac{\omega_1 f_2 + \omega_2 f_1}{f_1 f_2} = x\frac{(\omega_1 + \omega_2)}{f_2 f_1}$$

$$x = \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2} \text{------------------(4)}$$

When the two lenses are made of same material i.e., $\omega_1 = \omega_2 = \omega$. Then

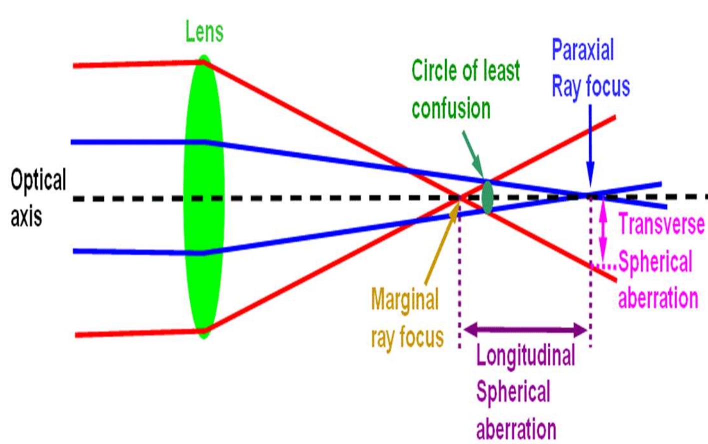
$$x = \frac{\omega f_2 + \omega f_1}{\omega + \omega} = \frac{\omega f_2 + \omega f_1}{2\omega} \qquad \Rightarrow \qquad \boxed{x = \frac{f_1 + f_2}{2}} \text{------------(5)}$$

The seperation between the two lenses must be equal to the mean of focal lengths of the two lenses.

SPHERICAL ABERRATION

The rays of light from the distant object after passing through the lens at the margin of the lens [known as marginal rays] converge at a point I_m close to the lens. Similarly, the rays of light passing through a region close to the axis [known as paraxial rays] converge at a point I_p, away from the lens. This results in an image that spreads into a region from I_m to I_p along the axis and from A to B perpendicular to the axis. This defect of the image due to the rays passing through different section of the lens, even with monochromatic light, is known as **spherical aberration of the lens.**

The spread of the image along the axis, [dx] is known as **longitudinal spherical aberration.** The image formed at AB is a circle with least diameter and at this position the best image is formed. This circle is called **the circle of least confusion.** The radius of the circle of least confusion measures **Transverse spherical aberration.**



Note: The spherical aberration in a convex lens is taken to be positive as the marginal image is formed near the lens than the paraxial image. In the case of concave lens, the spherical aberration is taken to be negative as the marginal image is formed to the right side of the paraxial image.

SPHERICAL ABERRATION BY PLANE REFRACTING SURFACE

Consider a plane surface AB perpendicular to *X*-axis and separating the two media as shown in fig. Let the refractive indices of two sides be μ_1 and μ_2 such that $\mu_2 > \mu_1$. Suppose *P* be an object point on the X-axis at a distance $-x_0$ from the origin *O*. In the figure PQ is a ray incident on the surface AB at height h from *O*. *i* is the angle of incident. QR is the refracted ray in denser medium. The refracted ray appears to emerge from a point P_1 on the axis. Hence, P_1 is the image of *P*. The distance of P_1 from *O* is $-x_1$

From $\triangle P_1QO$, $\tan r = \dfrac{h}{-x_1}$ or $\dfrac{\sin r}{\cos r} = \dfrac{h}{-x_1} \Rightarrow x_1 = -h\dfrac{\cos r}{\sin r} \Rightarrow x_1 = -\dfrac{-h\sqrt{(1-\sin^2 r)}}{\sin r}$

$x_1 = \dfrac{-h}{\sin r}\sqrt{1-\dfrac{\sin^2 i}{\mu^2}}$ -----> **(1)** $\left(\because \dfrac{\sin i}{\sin r} = \dfrac{\mu_2}{\mu_1} = \mu \Rightarrow \sin r = \dfrac{\sin i}{\mu}\right)$

From fig $\sin i = \dfrac{h}{\sqrt{h^2+x_0^2}}$ --------> **(2)**

Substituting, the value of **sin *i*** from eq (2) in eq (1) we get

$$x_1 = \frac{-h}{\sin i/\mu}\sqrt{1-\frac{\left(h/\sqrt{h^2+x_0^2}\right)^2}{\mu^2}} \Rightarrow \frac{-h}{\left(h/\sqrt{h^2+x_0^2}\right)/\mu}\sqrt{1-\frac{\left(h/\sqrt{h^2+x_0^2}\right)^2}{\mu^2}}$$

$$x_1 = \frac{-\mu h}{h}\sqrt{\left(h^2+x_0^2\right)}\sqrt{\left(1-\frac{h^2}{\mu^2\left(h^2+x_0^2\right)}\right)} \Rightarrow x_1 = -\mu x_0(1+\frac{h^2}{x_0^2})^{1/2}\left\{1-\frac{h^2}{x_0^2}\left((1+\frac{h^2}{x_0^2})^{-1}\right)\right\}^{1/2}$$

$$x_1 = -\mu x_0(1+\frac{h^2}{2x_0^2})\left\{1-\frac{h^2}{\mu^2 x_0^2}\left(1-\frac{h^2}{x_0^2}\right)\right\}^{1/2}$$ (using binomial expansion)

$$x_1 = -\mu x_0(1+\frac{h^2}{2x_0^2})\left\{1-\frac{h^2}{\mu^2 x_0^2}+\frac{h^4}{\mu^2 x_0^4}\right\}^{1/2} \Rightarrow x_1 = -\mu x_0(1+\frac{h^2}{2x_0^2})\left\{1-\frac{h^2}{\mu^2 x_0^2}\right\}^{1/2}$$ (neglecting higher terms h^4)

$x_1 = -\mu x_0 (1+\frac{h^2}{2x_0^2})\left\{1-\frac{h^2}{2\mu^2 x_0^2}\right\} \Rightarrow -\mu x_0\left(1+\frac{h^2}{2x_0^2}-\frac{h^2}{2\mu^2 x_0^2}-\frac{h^4}{2\mu^2 x_0^4}\right)$(neglecting higher terms h^4 and taking common $\frac{h^2}{2x_0^2}$)

$$\Rightarrow -\mu x_0\left\{1+\frac{h^2}{2x_0^2}(1-\frac{1}{\mu^2})\right\} \Rightarrow -\mu x_0 - \frac{h^2\mu}{2x_0}(1-\frac{1}{\mu^2}) \text{-----> } \mathbf{(3)}$$

For Paraxial rays, i.e., in the limiting case when h→0, eq (3) reduces $x_1 = -\mu x_0$ -----> **(4)**

Equations (3) and (4)

Spherical deviation = $-\frac{h^2\mu}{2x_0}(1-\frac{1}{\mu^2})$

The negative sign shows that the non-paraxial rays appear to diverge from a point which is still further away from the paraxial image point.

SPHERICAL ABERRATION DUE TO SPHERICAL SURFACE

AB is a spherical surface of radius of curvature *R*. Let an incident ray parallel to the principal axis meets the spherical surface at a height *h*, from the axis. The refracted ray intersects the axis at a point F_h. hence OF is equal to the focal length f_h

From Figure $f_h = R + CF_h$ ------> **(1)**

Consider △CPF_h

$$\frac{CF_h}{\sin r} = \frac{R}{\sin(i-r)} \Rightarrow CF_h = \frac{R\sin r}{\sin(i-r)}$$

$$CF_h = \frac{R\sin r}{\sin i\cos r - \cos i\sin r} = \frac{R\sin r}{\mu\sin r\cos r - \cos i\sin r} (\because \mu = \sin i / \sin r)$$

$$= \frac{R}{\mu\cos r - \cos i} \text{--------> } \mathbf{(2)}$$

Substituting the value of CF_h from eq (2) in eq (1) we get

$$f_h = R + \frac{R}{\mu\cos r - \cos i} \Rightarrow f_h = R\left[1+\frac{1}{\mu\cos r - \cos i}\right] \text{--------> } \mathbf{(3)}$$

For paraxial rays, h→0 ∴ cos *i* and cos *r* →1. In this case $f_p = \mu R / (\mu - 1)$ ------>**(4)**

The change in focal length for h zone as compared to axial zone is given by

$$\Delta f_h = f_p - f_h = \mu R/(\mu-1) - R\left[1+\frac{1}{\mu\cos r-\cos i}\right] \Rightarrow R\left[\frac{\mu}{\mu-1} - 1 - \frac{1}{\mu\cos r - \cos i}\right]$$

$$= R\left[\frac{1}{\mu-1} - \frac{1}{\mu\cos r - \cos i}\right] \text{---------> } \mathbf{(5)}$$

This represents the longitudinal spherical aberration. The approximate value of spherical aberration can be calculated as

From figure sin i = h/R ∴ sin r = h/μR (∵ sin i/sin r = μ)

$$\cos i = \sqrt{1-\sin^2 i} = (1-h^2/R^2)^{1/2} = 1-h^2/2R^2$$

$$\cos r = \sqrt{1-\sin^2 r} = \left(1-h^2\big/\mu^2R^2\right)^{1/2} = \left(1-h^2\big/2\mu^2R^2\right)$$

Substituting the values in (5) we get

$$\Delta f_h = R\left[\frac{1}{\mu-1} - \frac{1}{\mu(1-h^2/2\mu^2R^2)-(1-h^2/2R^2)}\right]$$

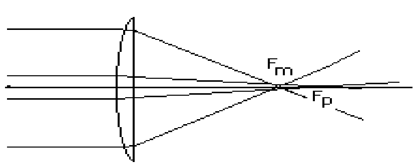
Solving we get $\boxed{\Delta f_h = \dfrac{h^2}{2(\mu-1)^2 f_p}}$ where $fp = \dfrac{\mu R}{(\mu-1)}$ ------------ > **(6)**

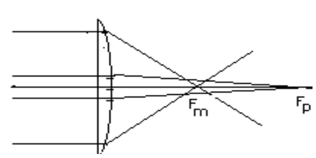
Equation (6) gives the approximate value of spherical aberration due to a spherical surface.

***Methods of reducing spherical aberration*:**

(1) By using stops: In this case, the stops used will either allow the paraxial rays or marginal rays. Usually, the stop is used to avoid the marginal rays. This brings paraxial and marginal images close to one another thereby reducing the spherical aberration.

(2) By the use of Plano-convex lens: In a lens, the deviation produced by the lens is minimum, when the deviation is shared equally between the two surfaces of the lens. This is achieved in a Plano-convex lens by arranging convex side facing the incident or emergent rays whichever are more parallel to the axis as shown in the following figure (2)





(3) By the use of crossed lenses: It is theoretically known that the lenses have minimum spherical aberration when the parallel rays fall of the lens having their radii of curvature r_1 and r_2 bearing a ratio, which satisfies the following condition. $\dfrac{r_1}{r_2} = \dfrac{\mu(2\mu-1)-4}{\mu(2\mu+1)}$

In the above equation, μ is the refractive index of the material of the lens. For a lens of μ = 1.5, the ratio $r_1/r_2 = -1/6$. A lens having its radii of curvature satisfying this condition is known as a ***crossed***

lens.

(4) **By using two Plano-convex** lenses separated by a suitable distance: When the two plano-convex lenses are separated at a suitable distance, the total deviation is divided equally between the two lenses and the total deviation is minimum. This reduces the spherical aberration to minimum.

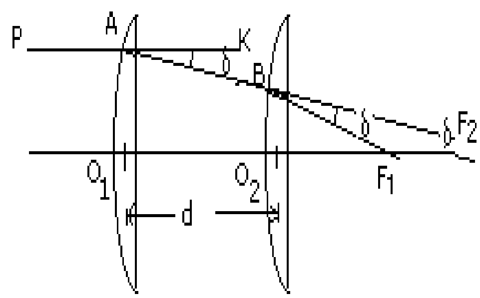
The necessary condition is derived as follows.

With reference to figure (4), we can write,

$\angle BAK = \angle BF_2O_2 = \delta$, Also, $\angle F_1BF_2 = \angle BF_2F_1 = \delta$

So that $F_1F_2 = F_1B = F_1O_2$ Or $O_2F_1 = \frac{1}{2} O_2F_2$

Since F_2 is the virtual object of the real image F_1 and using the lens formula for the second lens, we can write the equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_2}$ -----> (1)



In this equation $u = f_1 - d$ & $v = \frac{f_1 - d}{2}$ ------> (2)

Substituting for 'u' and 'v' and simplifying, we get

$$\frac{2}{f_1 - d} - \frac{1}{f_1 - d} = \frac{1}{f_2} \Rightarrow$$

$$\frac{1}{f_1 - d} = \frac{1}{f_2} \quad \text{--------> (3)}$$

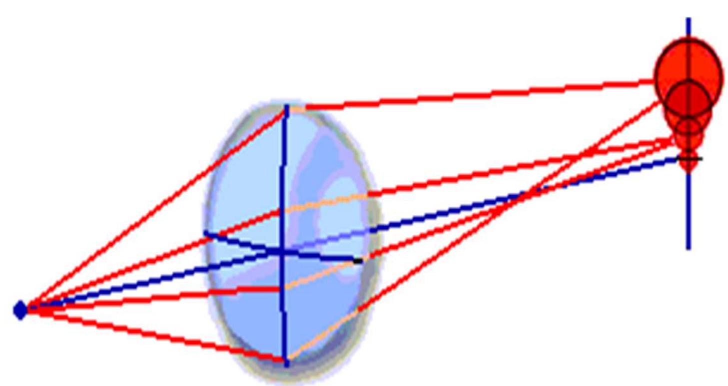
$$\Rightarrow f_2 = f_1 - d$$

Or $d = f_1 - f_2$ ------> (4)

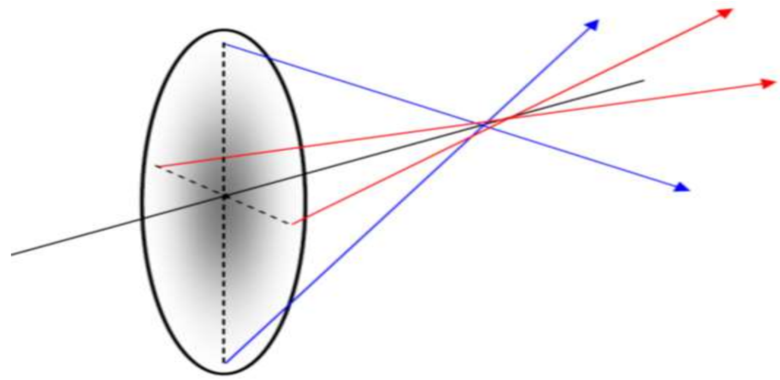
Eq (4) gives the condition for minimum spherical aberration

(5) **By using suitable concave and convex lenses in contact**: Since spherical aberration produced by convex lens is positive and that produced by a concave lens is negative, a suitable combination of convex and concave lens will minimize the spherical aberration.

<u>COMA</u> *When the point object is situated slightly off the axis, the image of the point object formed by the lens is found to have an egg-like (or comet like) shape*. The defect in the image is called **coma** after its shape.



Minimisation:



1. Coma can be reduced to a certain extent by the use of proper stop placed at a suitable distance from the lens.

2. Coma may be minimized by designing lenses of suitable shapes and materials.

For Ex. A lens with μ=1.5 and $R_1/R_2=-1/9$ forms an image of an object at finite distance sufficiently free from Coma.

3. Abbe showed that lenses are completely free from spherical aberration and coma by the following sine condition is satisfied. $\mu_1 y_1 \sin\theta_1 = \mu_2 y_2 \sin\theta_2$

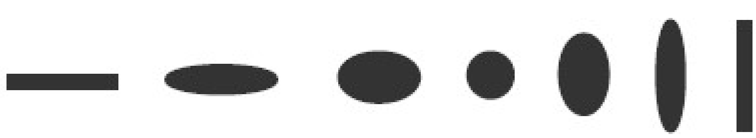
where μ_1, y_1, and $\sin\theta_1$ refers to the refractive index, height of the object above the axis and the slope angle of the incident ray of light respectively. Similarly, μ_2, y_2, and $\sin\theta_2$ refers to the corresponding quantities in the image space. The magnification of the image is given by y_2/y_1. When the medium on both sides is the same, then the above condition reduces to

$$\frac{y_2}{y_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

Elimination of the coma is possible if the lateral magnification y_2/y_1 is the same of all rays of light, irrespective of slope angles θ_1, θ_2. Thus, coma can be eliminated if $\sin\theta_1 / \sin\theta_2$ = constant, because μ_1/μ_2 is a constant. A lens that satisfies the above condition is called an aplanatic lens.

ASTIGMATISM: *Astigmatism is to imagine a lens (or mirror) whose surfaces are not exactly spherical but for which the radius of curvature (and hence focal length) in one plane is different from the radius of curvature in a plane at right angles to the first.*

As you move along the optical axis, the image changes from a horizontal line to a vertical line in a sequence that looks something like this:



Somewhere about halfway between the two linear images is the “circle of least confusion”.

Minimization

1. The astigmatism due to large inclination of the rays with the axis of the lens. Therefore, if the rays making large angles with the axis are cut off the defect can be eliminated. It can be reduced to a certain extent using proper stop placed at a suitable distance from the lens

2. Suitable convex and concave lens separated at a suitable distance may be used to reduce astigmatism.

Curvature of the field

The image of an extended object due to a single lens is not a flat one but it will be a curved surface. The central portion of the image nearer the axis is in focus but the outer regions of the image away from the axis are blurred. This defect is called the curvature of field.

Distortion

The Failure of a lens to form a point image due to a point object is due to the presence of spherical aberration, coma and astigmatism. *The variation in the magnification produced by a lens for different axial distances results in an aberration called Distortion*.

FIBER OPTICS

Introduction:- Fiber optics is the branch of science and engineering concerned with optical fibers.

In case of electromagnetic communication, the information is normally carried out in the form of radio waves and microwaves, through copper wires and co-axial cables. However, the information carrying capacity of these wires is restricted, due to their limited bandwidth and is not sufficient, as per the needs of modern communication techniques.

If light waves are used instead of radio waves, or microwaves, the number of signals transmitted can be increased enormously.

By using copper or aluminium wires one can at most send 48 simultaneous telephone conversations where as in an optical communication system transmission of about 15 million simultaneous telephone conversations through one hair thin optical fiber is possible.

The first useful optical fiber (doping of Silica glass with titanium) was invented in 1970 by Maurer, Keck, Schultz and Zimar.

Advantages of Optical fibers over wires:

1) Low loss
2) Large data-carrying capacity (Thousands of times greater)
3) High electrical resistance, so safe to use near high voltage equipment or between area with different earth potentials.
4) Light weight and
5) Signals contain very little power.

Disadvantages of Optical fibers over wires:

1) Higher cost
2) Need for more expensive optical transmitters and receivers.
3) More difficult and expensive to splice than wires and
4) Cannot carry electrical power to operate terminal devices.

Optical fibers:

Principle: The optical fibers are based on the principle of **total internal reflection* (TIR).

* <u>Total Internal Reflection (TIR):</u> - If a light ray is incident at the interface of a denser medium ($n_2 > n_1$), the refracted ray will bend towards the normal. On the other hand, if a ray is incident at the interface of a rarer medium ($n_2 < n_1$) then the ray will bend away from the normal. The angle of incidence, for which the angle of refraction is 90^0,is known as the *critical angle* and is denoted by ϕ_c. Thus, when $\phi_1 = \phi_c = \text{Sin}^{-1} (n_1/ n_2)$ --- (1)

$\phi_2 = 90^0$. When the angle of incidence exceeds the critical angle (i.e., when $\phi_1 > \phi_c$), there is no refracted ray and we have what is known as total internal reflection (TIR).

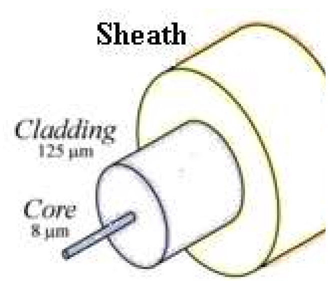
<u>Definition:-</u> An optical fiber is a hair thin cylindrical tube of glass or any transparent dielectric medium for transmitting light.

It is designed to guide light waves along the length of the fiber with the help of successive total internal reflections from sidewalls of the fiber.

<u>Construction</u>: - An optical fiber consists, in general, the following three regions:

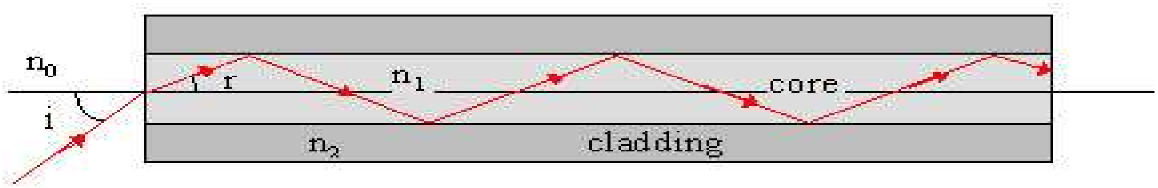
i) Innermost region, which is the light guiding region, called core. It is made of pure Silicon dioxide (SiO_2)
ii) Core region is surrounded by a middle region called Cladding. The refractive index of cladding ($n_2 \approx 1.45$) is always lower than that of the core ($n_1 \approx 1.465$). The cladding is usually pure silica while the core is usually silica doped with germanium.
iii) The outermost region is called sheath (made of Plastic or Teflon). The sheath protects the core and cladding from moisture, abrasion contamination and to give mechanical strength to fiber.

Basic purpose of cladding is to confine the light to the core. As the light falling on core and striking core cladding interface at an angle greater than the critical angle will be reflected back to the core.



Main function of the optical fiber is to accept maximum light and transmit the same with minimum attenuation or reduction.

Propagation of light through optical fiber:



Consider the figure, which is a section of the optical fiber. The refractive indices of the core and the cladding are n_1 and n_2 respectively. The fiber is normally in air (n_0=1) but could also be in a medium of refractive index n_0. The axis of the cylindrical structure is the optical axis. A ray is incident at an angle ' i 'at the entrance face and refracted into the core. It then strikes the core-cladding interface at a certain angle. If this angle exceeds the critical angle, it is totally reflected and strikes the interface on the other side of the axis. Here it is again totally reflected. This process is repeated till the ray emerges out of the fiber at the other end. The ray is thus guided by total internal reflection.

The angle of incidence at the entrance face for which the ray strikes the core-cladding interface at the critical angle is called the *Acceptance angle* or cut-off angle. The ray is guided for all the angles of incidence smaller than the acceptance angle at the entrance face. However, if the angle of incidence at the interface is less than the critical angle, both reflection and refraction takes place. Due to refraction at each incidence on the interface, the light beam dies off over a certain distance. There is no guidance.

Types of optical fibers: In an optical fiber light travels as an electromagnetic wave and all the waves moving in directions above the critical angle will be trapped in the fiber due to TIR. However all such waves do not propagate through the fiber, and only certain ray directions are allowed for propagations. These allowed directions correspond to modes of fiber.

The light ray paths along which the waves are in phase inside the fiber are called modes. Number of modes, a fiber can support depends on the ratio d/λ where d is the diameter of the core and λ is the wavelength of the wave transmitted.]

Optical fibers are in general of two types:

i) Single Mode Fiber (SMF) and ii) Multi Mode Fiber (MMF)

A ***single mode*** fiber has a smaller core diameter (<10μm) and can support only one mode of propagation.

For intercity cabling (i.e., for long distances) and highest speed Single mode fibers are used.

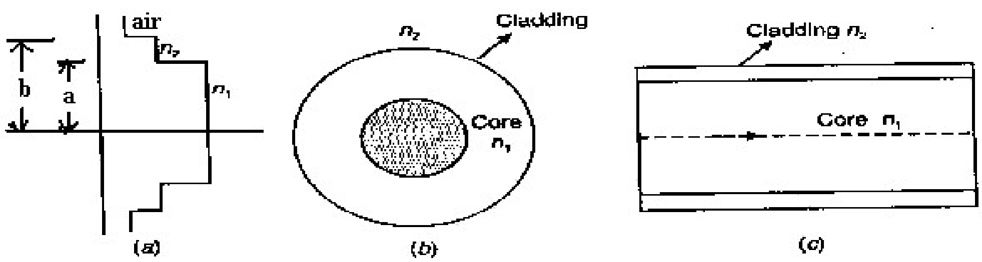
Advantages: Low loss, data rate upto 40 GB/Sec.

Multi mode *fiber* (MMF) has a large core diameter (50µm to 100µm) and can support a large number of modes.

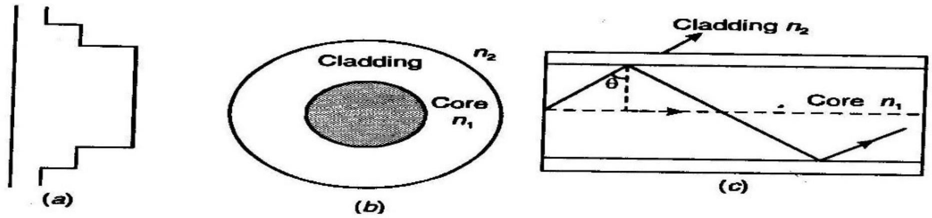
For short distances MMF's are used.

Multi mode fibers are further classified on the basis of index profile. [Index profile is a graph of refractive index (along X-axis) and distance from the core (along Y-axis)]. Index profile of a MMF can be either step index (SI) type or graded index (GRIN) type. Index profile of SMF is usually a step index type.

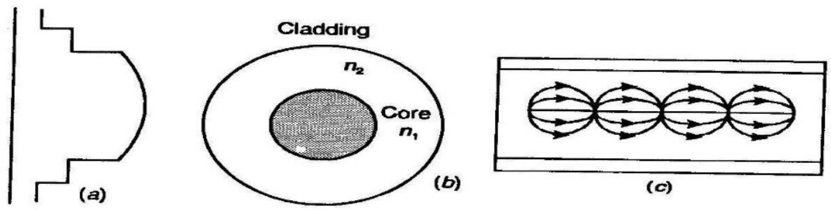
Single mode step index fiber – It consists of a very fine thin core (of radius 'a') of uniform refractive index surrounded by a cladding (of radius 'b') of refractive index lower than that of the core. Since the refractive index abruptly changes at the core-cladding boundary, it is known as step index fiber. A typical SMF has a core diameter of 4µm. Light travels along a single path.



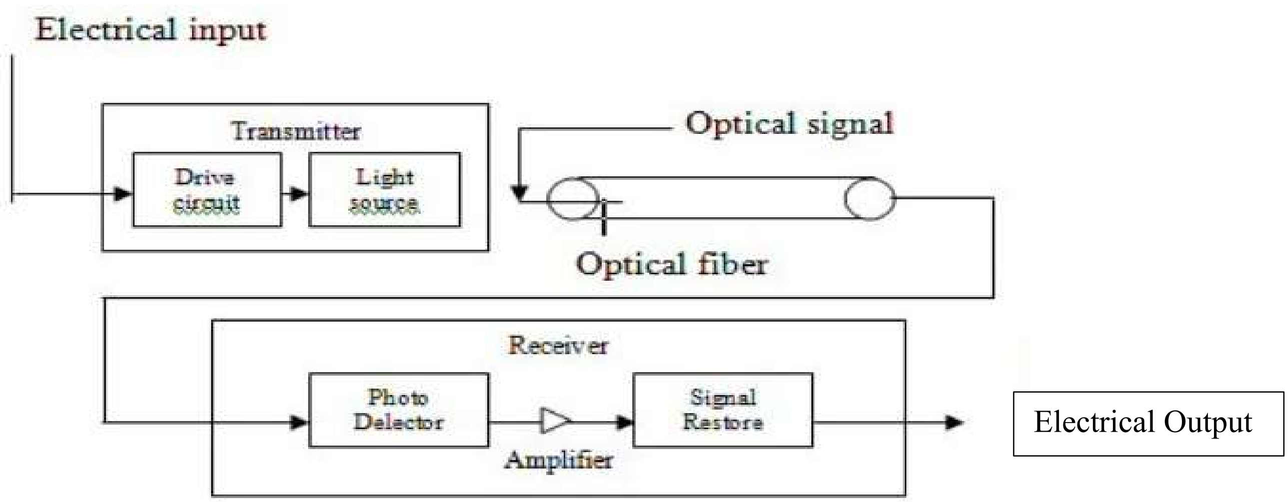
Multi mode step index fiber – It is similar to the single mode step index fiber with the exception that it has a large diameter (~ 100 µm). Core diameter is very large as compared to the wavelength of transmitted light. Light moves in zigzag paths along MMF. Typical structure along with profile of step MMF is shown in figure.



Graded index fiber (GRIN) – A GRIN is a multi mode fiber, which has concentric layers of refractive indices, which means that the refractive index of the core varies with distance from the fibers axis. i.e., it has high value at the center and falls of rapidly as the radial distance increases from the axis. In case of GRIN fibers, acceptance angel and numerical aperture diminish with increase of radial distance from the axis.



Basic principle of Optical fibre communication



At the transmitting end, the transmitted information (such as voice) is first converted into an electrical signal, and then modulated onto the laser beam emitted by the laser, so that the intensity of the light changes with the amplitude (frequency) of the electrical signal and passes through the optical fiber. The principle of total reflection is transmitted; at the receiving end, after receiving the optical signal, the detector converts it into an electrical signal, and after demodulation, restores the original information.

Optical communication utilizes the principle of total reflection. When the injection angle of light satisfies certain conditions, light can form total reflection in the optical fiber, thereby achieving the purpose of long-distance transmission. The light guiding properties of an optical fiber are based on total reflection of the light ray at the core and cladding interfaces, limiting light transmission in the core. There are two types of light in the fiber, namely meridional and oblique rays. The meridional rays are the light rays on the meridional plane, and the oblique rays are the light that is transmitted through the fiber axis.

<u>*Unit V - LASERS*</u>

A **LASER** (**L**ight **A**mplification by **S**timulated **E**mission of **R**adiation) is an optical source that emits photons in a coherent beam..

The process of particle transfer from normal state corresponding to minimum energy of the system to a higher energy state is termed as *excitation* and the particle itself is said to be excited. In this process the absorption of energy from the external field takes place.

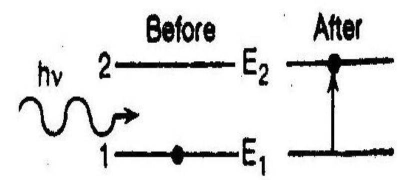
The time during which a particle can exist in the ground state is unlimited. On the other hand, the particle can remain in the excited state for a limited time known as *lifetime*.

The lifetime of the excited hydrogen atom is of the order of 10^{-8}sec. Their exist, some excited states in which the lifetime is >10^{-8} sec. These states are called as *metastable*.

The basic principle involved in laser action is the phenomenon of <u>*stimulated emission*</u>.

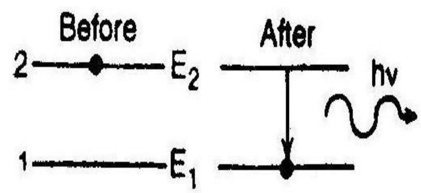
There are three kinds of electromagnetic radiations between two energy levels E1 and E2 in an atom.

<u>*i) Induced absorption*</u>:- If the atom is initially in the lower state E1 it can be raised to E2 by absorption of a photon of energy E2- E1 = hν . This is called *induced absorption*.



After being in the excited state, the particle returns to the ground state.

<u>*ii) Spontaneous emission:-*</u> If the atom is initially in the upper state E2, it can drop to E1 by emitting of a photon of energy hν. This process is known as *Spontaneous emission*.

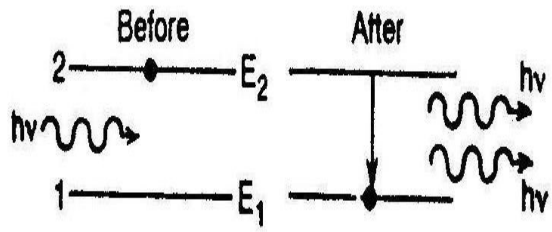


The spontaneous emission depends on the type of the particle and type of transition but is independent of outside circumstances.

The waves coincide neither in wavelength nor in phase. Thus the radiation is incoherent and has a broad spectrum.

The rate of spontaneous emission is proportional to the number of atoms in the excited state.

iii) Stimulated emission:- If an atom is already in the excited state of energy level E_2 whose ground level energy is E_1, at this moment, a photon of energy $h\nu = E_2 - E_1$ is incident on the excited atom, the incident photon stimulates a similar photon from the excited atom. Now the atom returns to the ground state.



This type of emission is called as *stimulated emission*.
It is coherent with the stimulating incident radiation. It has the same frequency & phase as the incident radiation.

The rate of stimulated emission depends both on the intensity of external field and also on the number of atoms in the excited state.

Differences between *spontaneous* and *stimulated emission:*

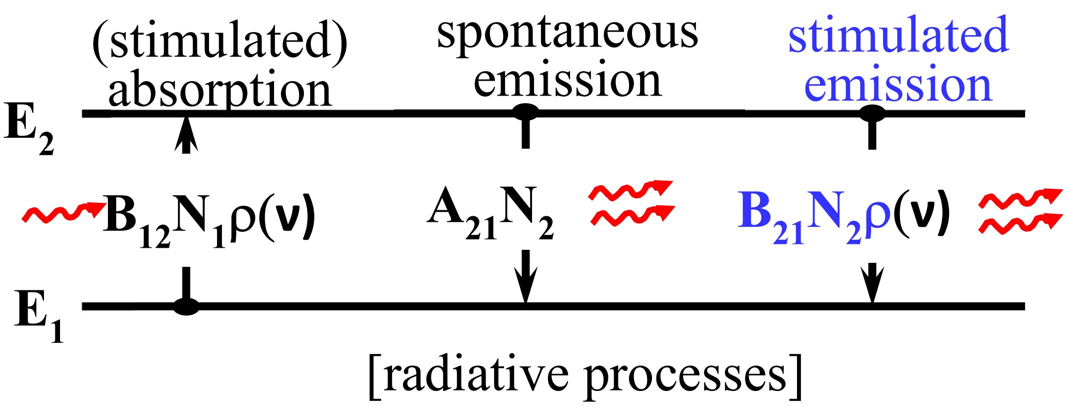
Spontaneous emission	Stimulated emission
1. Transition occurs from a higher energy level to a lower energy level.	Transition also occurs from higher energy level to lower energy level.
2. No incident photon is required	Photon whose energy is equal to the difference of two energy levels is required.
3. Single photon is emitted.	Two photons with same energy are emitted.
4. The energy of emitted photon is equal to the energy difference of two levels.	The energy of the emitted photons is double the energy of stimulated photons.
5. This was postulated by Bohr.	This was postulated by Einstein.

Population Inversion: The number of particles N_2, i.e., population of higher energy level is less than the population N_1 of lower energy level. Making the number of particles N_2 more in higher energy level than the number of particles N_1 in lower energy level ($N_2 > N_1$) is called as population inversion or inverted population.

A system in which population inversion is achieved is called an active system. *The method of raising the particle from lower energy state to higher energy state is called as pumping*. A more common method of pumping is optical pumping.

Einstein theory of Lasers (OR) Einstein's Coefficients:

Based on Einstein's quantum theory of radiation one can get the expression for probability for stimulated emission to the probability for spontaneous emission under thermal equilibrium. Consider two energy levels E_1 and E_2 of an atomic system such that $E_2 > E_1$. Let N_1 and N_2 be the number atoms per unit volume present at the levels E_1 and E_2 respectively. In any rotation of frequency corresponding to the energy difference (E_2-E_1) fall on the atomic system it can interact with the matter in ***three distinct ways***.



I. Absorption:

If $\rho(\nu)d\nu$ is the radiation energy per unit volume between the frequency range ν and $\nu + d\nu$, then the number of atoms undergoing in absorption per unit volume per second from levels E_1 to E_2 = $N_1\rho(\nu)B_{12}$ -------------------- (1)

Where B_{12} is the probability of absorption per unit time.

II. ***Stimulated Emission:***

When an atom males trasition from E_2 to E_1, in the presence of external photon whose energy is equal to (E_2-E_1) stimulated emission takes place. Thus the number of stimulated emissions per unit volume per second and from levels E_2 to E_1= $N_2\rho(\nu)B_{21}$ ------------------ (2)

where, B_{21} represents probability of stimulated emission per unit time.

III. ***Spontaneous emission:***

An atom in the level E_2 can also make a spontaneous emission by jumping into lower energy level E_1 the number of atoms making spontaneous emission per unit volume per second from levels E_2 to $E_1 = N_2 A_{21}$ ---------------------------- (3)

where A_{21} is represents probability of spontaneous emission per unit time.

Under steady state condition $\frac{dN}{dT} = 0$

$\therefore$ Number of atoms undergoing absorption per second = Number of atoms undergoing emission per second.

$\therefore$ Eq (1) = Eq (2) +Eq(3)

$$N_1 \rho(\nu) B_{12} = N_2 \rho(\nu) B_{21} + N_2 A_{21}$$

$$\therefore \rho(\nu) = \frac{A_{21}}{(\frac{N_1}{N_2}) B_{12} - B_{21}} \quad \text{-------------------------} \quad (4)$$

From Boltzman distribution law

$$\frac{N_1}{N_2} = e^{(E_1 - E_2)/k_B T} = e^{h\nu/kT} \quad \text{----------------------------} \quad (5)$$

Sub $\frac{N_1}{N_2}$ in eq (4) and using $B_{12} = B_{21}$, we get

$$\rho(\nu) = \frac{A_{21}}{B_{21}(e^{h\nu/kT} - 1)} \quad \text{---------------------} \quad (6)$$

From Planck's radiation law we have,

$$\rho(\nu) = \frac{8\pi h n^3}{\lambda^3} \frac{1}{(e^{h\nu/kT} - 1)} \quad \text{----------------------} \quad (7)$$

where n = refractive index of the medium

λ = wavelength of light in air.

Let λ_m be the wavelength of the medium,

$$\lambda_m = \lambda / n \qquad \text{------------------------} \quad (8)$$

$$\rho(\nu) = \frac{8\pi h}{\lambda_m^3} \frac{1}{(e^{h\nu/kT} - 1)} \qquad \text{-------------------------} \quad (9)$$

Comparing eq (6) and (9)

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h}{\lambda_m^3}$$

A_{21} and B_{21} are called Einstein's Coefficients of spontaneous emission probability per unit time and stimulated emission probability per unit time respectively.

For stimulated emission to be predominant we need,

$$\frac{A_{21}}{B_{21}} << 1$$

The function $\frac{1}{(e^{h\nu/kT} - 1)}$ represents the ratio of stimulated emission rate to spontaneous emission rate.

Characteristics of a laser:

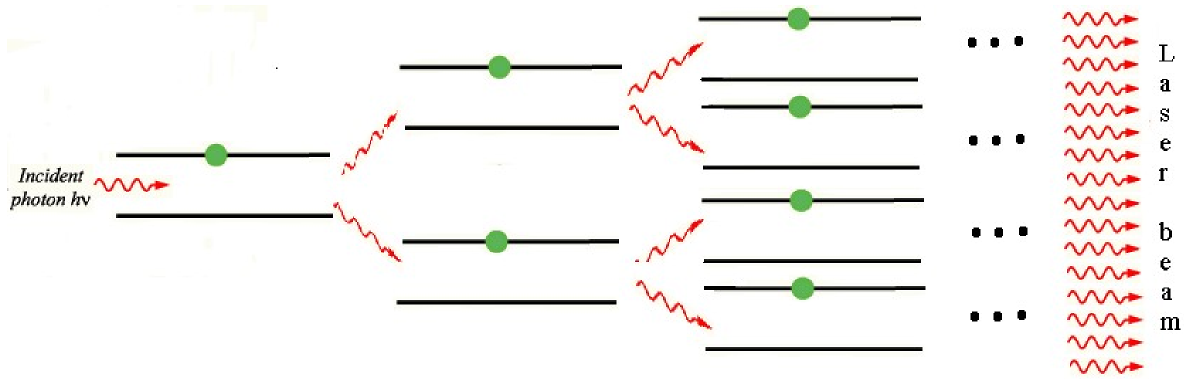
i) *The light is coherent* with all the waves exactly in phase with each other.

ii) *Laser beam hardly diverges*. i.e., The laser rays are almost parallel.

iii) *The beam is nearly monochromatic*.

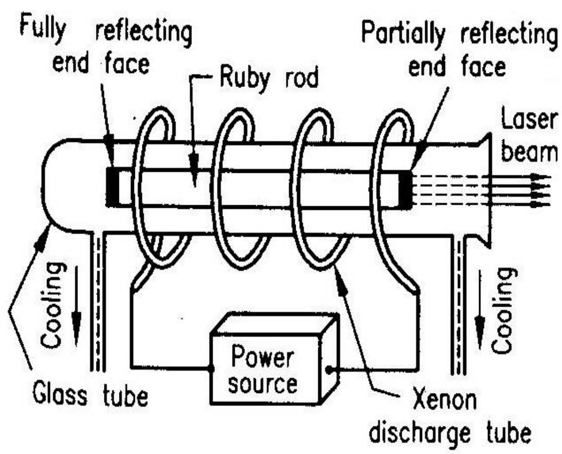
iv) *The laser beam is extremely intense*. The beam can produce a temperature of 10^4 ºC at a focused point.

Principle of operation of lasers: Consider a group of atoms all in the same excited state. A passing photon may cause stimulated emission in one of these atoms. This results in the emission of two photons. Each of these photons may cause induced emission in two other excited atoms. This process may continue in a chain reaction. The result will be an intense beam of photons moving in the same direction and all are coherent.



<u>Ruby laser:</u> The first successful laser utilized a ruby rod.

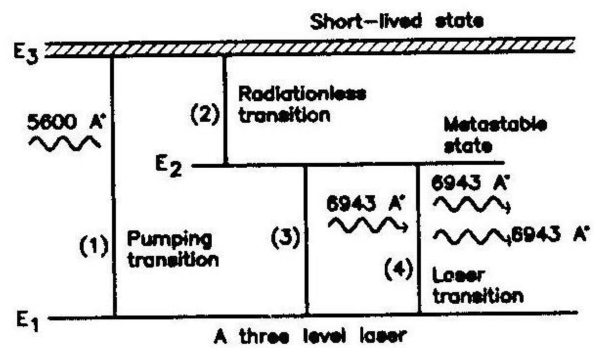
<u>Construction</u>:- A ruby is a crystal of aluminium oxide Al_2O_3 in which some aluminium atoms are replaced by chromium atoms (Cr_2O_3). The active materials in the ruby are chromium ions Cr^{3+}. In a ruby laser, a rod of 4cm length and 0.5cm in diameter is generally used. The two ends of the ruby rod are made perfectly parallel to each other.



One end A is heavily silvered and the other end B of the rod is partially silvered. The rod is surrounded by a helical xenon flash tube, which provides the pumping light to raise the chromium ions to upper energy level. Only a part of the energy is used in pumping the Cr^{3+} ions while the rest heats up the apparatus. For these purpose a cooling arrangement (liquid nitrogen) is used.

<u>Working:-</u> An energy diagram illustrating the operation principle of a ruby laser is shown in figure. In the fig. E_1, E_2 and E_3 represent the energy levels of chromium ion. In normal state, the chromium ion is in lower level. When the ruby crystal is irradiated with light of xenon flash, the chromium atoms are excited and pass to upper level. Few excited atoms return to ground level E_1 and other to level E_2. The transitions $E_3 \rightarrow E_2$ are non-radiative

. i.e., the chromium atoms give part of their energy to crystal lattice in the form of heat. After few milliseconds, the level E_2 becomes more populated than level E_1 and hence the desired population inversion is achieved. The spontaneous transition may cause an induced transition (stimulated emission), which produces a photon. Photon traveling parallel to the axis of the tube (crystal) will start a cascade of photon emission while the photons traveling in any direction other than this will pass out of ruby.



The ruby laser is an example of a *three level laser*.

The wavelength of out put beam is 6943 A^0, The duration of out put flash is 300 μ sec and the Intensity of out put beam is 10,000 watt.

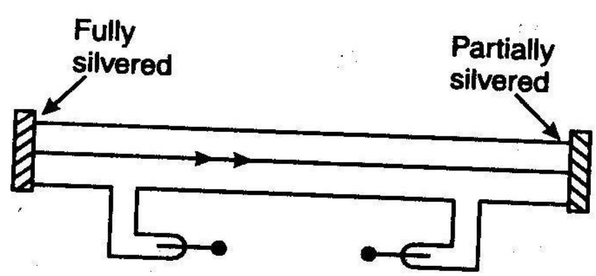
<u>*Drawbacks*</u>: i) The Ruby laser requires high pumping power

ii) It is a pulsed laser.

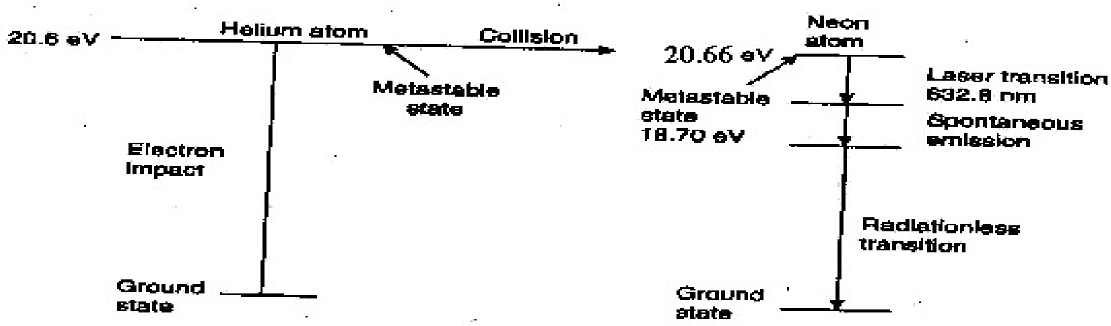
<u>He – Ne Laser:</u> The main drawback of Ruby laser is that the out put beam is not continuous though very intense. For the continuous laser beam, gas lasers are used. He- Ne laser (fabricated by Ali Javan and his associates in USA) is the first one to be operated successfully.

<u>Construction:</u> The laser tube is approximately 5mm in diameter and 0.5m long. It contains helium – neon mixture, in the ratio 5:1 at a total pressure of about 1mm of mercury.

The ends of the tube are plane and parallel. One end of the tube is heavily silvered and the other end is partially silvered. An electric discharge is produced in the gas mixture by electrodes connected to a high frequency electric source.



Working:



The collisions of the He & Ne atoms with the electrons from the discharge excite (or pump) the helium & neon atoms to metastable states. Some of the excited He atoms transfer their energy to ground state Ne atoms by collisions. Thus He atoms help in achieving a population inversion in the Ne atoms. When a Ne atom passes spontaneously from the metastable state at 20.66eV to state at 18.70eV, it emits photon. This photon travels through the gas mixture, and if it is moving parallel to the axis of the tube, is reflected back & forth by the mirror ends until it stimulates an excited Ne atom and causes it to emit a fresh photon in phase with the stimulating photon. This stimulated transition from 20.66eV level to 18.70eV level is the laser transition. This process is continued and a beam of coherent radiation builds up in the tube. When the beam becomes sufficiently intense, a portion of it escapes through the partially – silvered end. The excitation of He & Ne atoms occur all the time, unlike the pulsed excitation in ruby laser, the He-Ne laser operates continuously.

Uses of laser:

In consumer electronics, telecommunications, and data communications, lasers are used as the transmitters in optical communications over optical fiber and free space. They are used to store and retrieve data from compact discs and DVDs, as well as magneto-optical discs. Laser lighting displays (pictured) accompany many music concerts.

In science, lasers are employed in a wide variety of interferometric techniques, and for Raman spectroscopy. Other uses include atmospheric remote sensing, and investigation of nonlinear optics phenomena. Holographic techniques employing lasers also contribute to a number of measurement techniques. Lasers have also been used aboard scientific spacecraft.

In medicine, the laser scalpel is used for laser vision correction and other surgical techniques. Lasers are also used for dermatological procedures including removal of tattoos, birthmarks, and hair.

In industry, laser cutting is used to cut steel and other metals. Laser line levels are used in surveying and construction. Lasers are also used for guidance for aircraft. Lasers are used in certain types of thermonuclear fusion reactors.

In law enforcement the most widely known use of lasers is for lidar to detect the speed of vehicles. Military uses of lasers include use as target designators for other weapons; their use as directed-energy weapons is currently under research.

Basic Principle of Holography

An object is illuminated with a beam of coherent light [object beam]. Then every point on the surface of the object acts as a source of secondary waves. These secondary waves spread in all directions. Some of these waves can fall on a recording plate [holographic plate]. Simultaneously, another beam of same coherent light [reference beam] can fall on this holographic plate. In the holographic plate, both the beams combine, and interference pattern will be formed. This interference pattern is recorded on the holographic plate. The three-dimensional image of the object can be seen by exposing the recorded holographic plate [hologram] to coherent light. This is the principle of holography.

Applications of holography

1. The three-dimensional images produced by holograms have been used in various fields, such as technical, educational also in advertising, artistic display etc.

2. Holographic diffraction gratings: The interference of two plane wavefronts of laser beams on the surface of holographic plate produces holographic diffraction grating. The lines in this grating are more uniform than in case of conventional grating.

3. Hologram is a reliable object for data storage, because even a small broken piece of hologram contains complete data or information about the object with reduced clarity.

4. The information-holding capacity of a hologram is very high because many objects can be recorded in a single hologram, by slightly changing ...